

# Abbey Lane Primary School



## Mathematics Calculation Policy

## Mathematics calculation Policy

This document has been written in line with the National Curriculum for mathematics September – 2014.

The national curriculum for mathematics aims to ensure that all pupils:

- Become **fluent** in the fundamentals of mathematics, including through varied and frequent practice with increasingly complex problems over time, so that pupils develop conceptual understanding and the ability to recall and apply knowledge rapidly and accurately.
- **Reason mathematically** by following a line of enquiry, conjecturing relationships and generalisations, and developing an argument, justification or proof using mathematical language
- Can **solve problems** by applying their mathematics to a variety of routine and non-routine problems with increasing sophistication, including breaking down problems into a series of simpler steps and persevering in seeking solutions

### **Aims of this policy**

This policy outlines progression through calculation strategies for addition, subtraction, multiplication and division in line with the new National Curriculum commencing September 2014. A school wide policy helps to ensure consistency of approach, enabling children to progress stage by stage through models and representations they recognise from previous teaching, allowing for deeper conceptual understanding and fluency. As children move at the pace appropriate to them, teachers will be presenting strategies and equipment appropriate to children's level of understanding. However, it is expected that the majority of children in each class will be working at age-appropriate levels as set out in the National Curriculum 2014 and in line with school policy. Each of the four operations build on a solid understanding of place value, the connections between the four number operations and number sense, such as: whether they are odd or even, whether they are close to multiples of ten or if they are close together:

- Children need to use correct mathematical terminology in context and be able to verbalise their calculation strategies.
- Children need to make considered decisions as to the most appropriate methods to make mathematics more functional. They need to choose the most appropriate, fluent, efficient and accurate method to do a particular calculation.
- Children need to use concrete resources before they progress to pictorial and abstract representations. This CPA (concrete, pictorial and abstract) approach needs to be available to children throughout school, as and when necessary. Use of manipulatives (Numicon, Cuisenaire, dienes, HTO counters etc.) helps reinforce understanding and provides support when calculating mentally, mentally with jottings, using expanded methods and formal written methods. Use of the bar model, number lines and part:part whole diagrams are recommended.
- Children should progress between the stages working towards formal written methods (where appropriate), once they have mastered each stage. However, they should not be hurried and, after the method has been taught, children should still be able to make their preferred choice of the most appropriate, efficient and accurate method for them. Previous stages may need to be revisited to consolidate understanding when introducing a new strategy.

- As new methods of calculations are introduced, children should have the opportunity to examine them, alongside the method they have consolidated, to make connections between the methods and establish the similarities and differences between them.

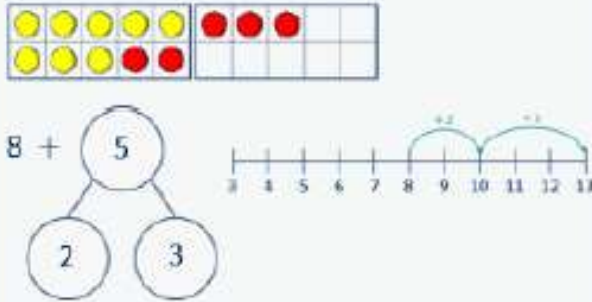
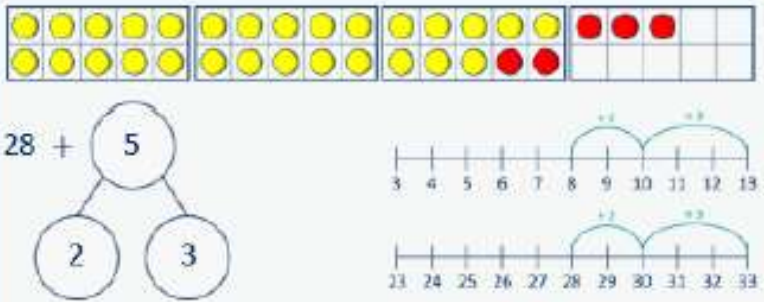
### Guidance for the use of the policy

The calculation policy is divided into four sections: addition, subtraction, multiplication and division. At the start of each section, you will find an overview of the progression of skills. Calculations involving decimal numbers and fractions are included.

The calculation policy follows the same concrete, pictorial, abstract approach as the White Rose main schemes of learning, which we use here at Abbey Lane. Where appropriate, sentence stems and key questions are included alongside the key representations.

Where skills are divided into more than one section across the page, there is a progression in the level of difficulty from left to right.

For example, when adding across a 10, children need to be able to add across 10 itself, before making links with related facts.

<p><b>Add across a 10</b></p> <p>Partition the number you are adding to make a full ten.</p>	<p>... can be partitioned into ... and ...</p> 	<p>I add ... to get to ... then I add ...</p> <p><math>8 + 5 = 13</math> <math>28 + 5 = 33</math></p> 
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### Mathematical Vocabulary

The 2014 National Curriculum reflects the importance of pupils using the correct mathematical language as a central part of their learning. ‘The quality and variety of language that pupils hear and speak are key factors in developing their mathematical vocabulary and presenting a mathematical justification, argument or proof’ (National Curriculum for Mathematics, 2014). It is therefore essential that teaching is accompanied by the use of correct vocabulary and that new vocabulary is introduced in a suitable context. Across school, teachers will refer back to the ‘Abbey Lane Mathematical Vocabulary Progression’ document to use a wide variety of mathematical vocabulary appropriate to each year group. This will also show progression between year groups so that teachers begin the year using vocabulary from the previous year, slowly building on that with new vocabulary.


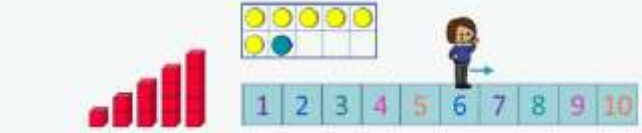



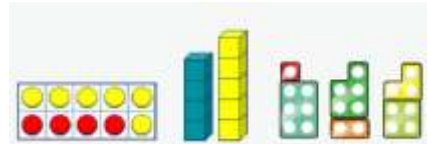

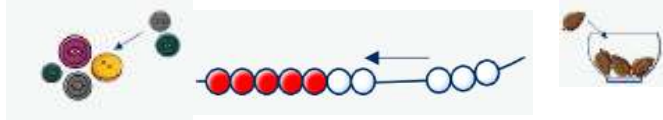
### Progression of skills


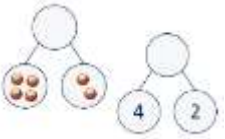
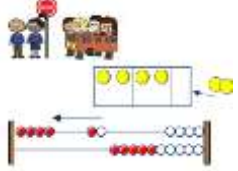
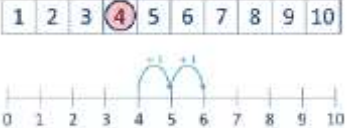
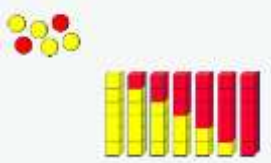

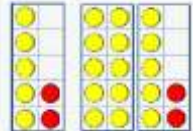
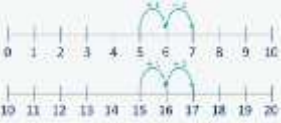

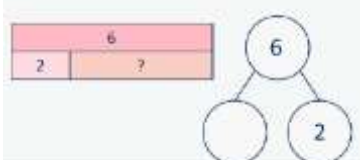
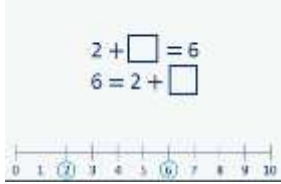
Building upon previous year’s learning is vital to provide a progressive and purposeful scheme of work to enable all learners to have the very best opportunities to succeed. The following demonstrates the progression made through the school years in the four calculation areas:

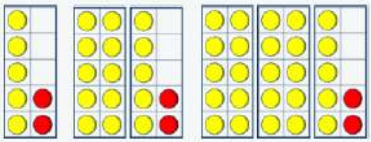
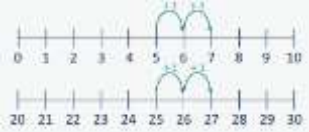
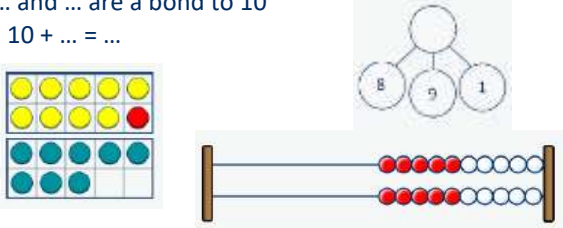
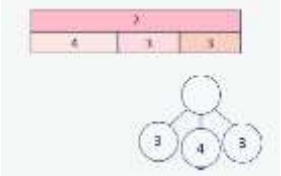
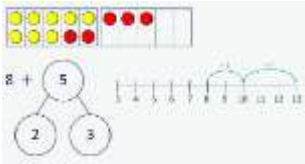
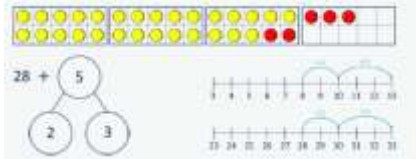
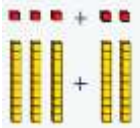

### Progression of skills - Addition

Year group	Skill
FS2	<ul style="list-style-type: none"> <li>• Conceptually subitise to 5</li> <li>• 1 more</li> <li>• Notice the composition of numbers within 10</li> <li>• Combine 2 groups</li> <li>• Add more</li> </ul>
Year 1	<ul style="list-style-type: none"> <li>• Add together</li> <li>• Add more</li> <li>• Bonds within 10</li> <li>• Related facts within 20</li> <li>• Missing numbers</li> </ul>
Year 2	<ul style="list-style-type: none"> <li>• Add 1s to any number (related facts)</li> <li>• Add three 1-digit numbers</li> <li>• Add across a 10</li> <li>• Add multiples of 10</li> <li>• Add 10s to any number</li> <li>• Add two 2-digit numbers (not across a ten)</li> <li>• Add two 2-digit numbers (across a ten)</li> <li>• Missing numbers</li> </ul>
Year 3	<ul style="list-style-type: none"> <li>• Add 1s, 10s and 100s to a 3-digit number</li> <li>• Add two numbers (no exchange)</li> <li>• Add two numbers across a 10 or 100</li> <li>• Complements to 100</li> <li>• Add fractions with the same denominator within 1 whole</li> <li>• Calculate the duration of events</li> </ul>
Year 4	<ul style="list-style-type: none"> <li>• Add 1s, 10s and 100s to a 4-digit number</li> <li>• Add up to two 4-digit numbers</li> <li>• Add decimal numbers in the context of money</li> <li>• Add fractions and mixed numbers with the same denominator beyond 1 whole</li> </ul>
Year 5	<ul style="list-style-type: none"> <li>• Add using mental strategies</li> <li>• Add whole numbers with more than 4 digits</li> <li>• Add decimals with up to 2 decimal places</li> <li>• Complements to 1</li> <li>• Add fractions with denominators that are a multiple of one another</li> </ul>
Year 6	<ul style="list-style-type: none"> <li>• Add integers up to 10 million</li> <li>• Add decimals with up to 3 decimal places</li> <li>• Order of operations</li> <li>• Negative numbers</li> <li>• Add fractions</li> </ul>

**Addition**

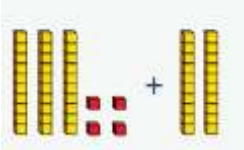
<p><b>FS2</b></p>	<ul style="list-style-type: none"> <li>• Have a deep understanding of numbers to 10, including the composition of each number.</li> <li>• Subitise (recognise quantities without counting) up to 5</li> <li>• Automatically recall (without reference to rhymes, counting or other aids) number bonds up to 5 and some number bonds to 10, including double facts.</li> </ul>	
<p><b>Progression of skills</b></p>	<p><b>Key representations</b></p>	
<p><b>Conceptually subitise to 5</b> Notice the parts that make up the whole.</p>	<p>What do you see? How do you see it?</p> 	
<p><b>1 more</b> Continue to link to stories, songs and rhymes.</p>	<p>1 more than ... is ...</p> 	
<p><b>Notice the composition of numbers within 10</b> Link to stories, songs and rhymes.</p>	<p>How many...? How many...? How many altogether?</p> 	<p>How many ways can you make...?</p> 
<p><b>Combine 2 groups</b> 2 groups are combined to find the total.</p>	<p>There are .... There are .... There are .... altogether.</p> 	<p>.... and .... make ....</p> 
<p><b>Add more</b> A quantity is increased.</p>	<p>First... Then.... Now....</p> 	<p>I have .... I add .... more. Now I have....</p> 

<b>Year 1</b>	<ul style="list-style-type: none"> <li>• Read, write and interpret mathematical statements involving addition (+) and equals (=) signs.</li> <li>• Represent and use number bonds within 20</li> <li>• Add 1-digit and 2-digit numbers to 20, including zero.</li> <li>• Solve one-step problems that involve addition, using concrete objects and pictorial representations, and missing number problems such as <math>7 = + 2</math></li> </ul>									
<b>Progression of skills</b>	<b>Key representations</b>									
<b>Add together</b> (aggregation) 2 quantities are combined to find the total.	<p>There are ... There are ... There are... altogether.</p> 	<p>... is a part. ... is a part. ... is the whole.</p> 	<p>... plus ... is equal to ... ... is equal to ... + ... <math>4 + 2 = 6</math> <math>2 + 4 = 6</math> <math>6 = 4 + 2</math> <math>6 = 2 + 4</math></p>							
<b>Add more</b> (augmentation) A quantity is increased.	<p>First... Then... Now...</p> 	<p>I start at ... I jump on ... I land on ...</p> 	<p>... plus ... is equal to ... ... is equal to ... + ... <math>4 + 2 = 6</math> <math>2 + 4 = 6</math> <math>6 = 4 + 2</math> <math>6 = 2 + 4</math></p>							
<b>Bonds within 10</b> Include bonds for each number within 10 Encourage children to notice patterns.	<p>... is made of ... and ... ... and ... make ...</p> 	<p>... can be partitioned into ... and ...</p> 	<p>... plus ... is equal to ...</p> <table border="0"> <tr> <td><math>3 + 3 = 6</math></td> </tr> <tr> <td><math>6 + 0 = 6</math></td> </tr> <tr> <td><math>2 + 4 = 6</math></td> </tr> <tr> <td><math>5 + 1 = 6</math></td> </tr> <tr> <td><math>1 + 5 = 6</math></td> </tr> <tr> <td><math>4 + 2 = 6</math></td> </tr> <tr> <td><math>0 + 6 = 6</math></td> </tr> </table>	$3 + 3 = 6$	$6 + 0 = 6$	$2 + 4 = 6$	$5 + 1 = 6$	$1 + 5 = 6$	$4 + 2 = 6$	$0 + 6 = 6$
$3 + 3 = 6$										
$6 + 0 = 6$										
$2 + 4 = 6$										
$5 + 1 = 6$										
$1 + 5 = 6$										
$4 + 2 = 6$										
$0 + 6 = 6$										
<b>Related facts within 20</b> Make links to known facts.	<p>I know that ... and ... = ... so ... and ... = ...</p> 	<p>... more than ... is ... so ... more than ... is ...</p> 	<p>What patterns do you notice?</p> <p><math>5 - 2 = 7</math> <math>15 - 2 = 17</math></p> <p><math>7 = 5 + 2</math> <math>17 = 15 + 2</math></p>							
<b>Missing numbers</b> Make links to known facts.	<p>How many more do you need to make ...?</p> 	<p>If ... is the whole and ... is a part, the other part must be...</p> 	<p>... plus ... is equal to ...</p> 							


<b>Year 2</b>	<ul style="list-style-type: none"> <li>Recall and use addition facts to 20 fluently, and derive and use related facts up to 100</li> <li>Add numbers using concrete objects, pictorial representations, and mentally, including: a two-digit number and 1s             <ul style="list-style-type: none"> <li>a two-digit number and 10s</li> <li>2 two-digit numbers</li> <li>adding 3 one-digit numbers</li> </ul> </li> <li>Recognise and use the inverse relationship between addition and subtraction and use this to check calculations and solve missing number problems.</li> </ul>		
<b>Progression of skills</b>	<b>Key representations</b>		
<b>Add ones to any number</b> (related facts) Make links to known facts.	I know that ... and ... = ... so ... and ... = ... 	... more than ... is ... so ... more than ... is ... 	What do you notice? Can you continue the pattern? $5 + 2 = 7$ $15 + 2 = 17$ $25 + 2 = 27...$
<b>Add three 1-digit numbers</b> Prompt children to understand that addition can be done in any order and to make links to known facts.	... and ... are a bond to 10 $10 + ... = ...$ 	Double ... + ... = ... 	What do you notice? Which addition is the easiest to calculate? $8 + 9 + 1 =$ $8 + 1 + 9 =$ $9 + 1 + 8 =$
<b>Add across a 10</b> Partition the number being added to make a full ten.	... can be partitioned into ... and ... 		I add ... to get to ... then I add ... <span style="float: right;"><math>8 + 5 = 13</math> <math>28 + 5 = 33</math></span> 
<b>Add multiples of 10</b> Make links to known facts within ten.	... ones + ... ones = ... ones so ... tens + ... tens = ... tens $3 + 2 = 5$ $30 + 20 = 50$ 		What is the same? What is different? 

**Add 10s to any number**  
 Make links to known facts.

... tens + ... tens = ... tens  
 ... tens and ... ones = ...



To add ... I need to add 10 ... times

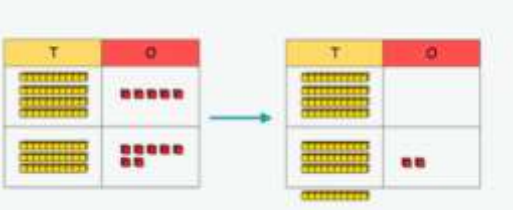
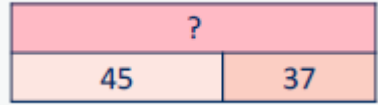
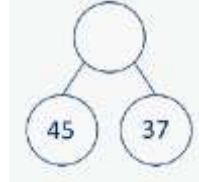


I know that ... and ... = ...  
 so ... and ... =

$30 + 20 = 50$   
 $34 + 20 = 54$

**Add 2-digit numbers**  
 (not across a ten)  
 Lining up ones and tens in columns will support with later written methods.


There are ... ones, so I do/do not need to make an exchange.  
 ... ones = ... ten and ... ones

5 ones + 7 ones = 12 ones  
 12 ones = 1 ten and 2 ones  
 4 tens + 3 tens + 1 ten =  
 8 tens 8 tens and 2 ones = 82

**Missing Numbers**  
 Solve missing number problems and use the inverse to check

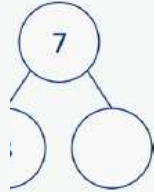
How many more do you need to make ...?



$6 + \square = 10$   
 $10 - \square = 6$

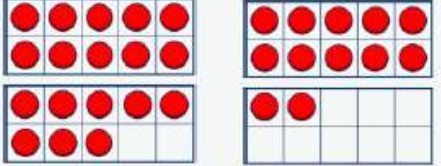
If ... is a whole and ... is a part, then ... is the other part.

$\square + 3 = 7$   
 $7 - 3 = \square$



can be partitioned into ... and ...

$10 + 8 = 12 + \square$



<b>Year 3</b>	<ul style="list-style-type: none"> <li>• Add numbers mentally, including: a three-digit number and ones, a three-digit number and tens, a three-digit number and hundreds.</li> <li>• Add numbers with up to three digits, using formal written methods of columnar addition.</li> <li>• Add fractions with the same denominator within 1 whole.</li> <li>• Calculate the time taken by particular events or tasks.</li> </ul>
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<b>Progression of skills</b>	<b>Key representations</b>
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**Add 1s, 10s or 100s to a 3-digit number**  
Emphasis on mental strategies including number bonds and related facts. Prompt children to notice which digit changes.

The ones/tens/hundreds column will increase by ...

$444 + 5 =$   
 $444 + 50 =$   
 $444 + 500 =$

$777 + 2 =$   
 $777 + 20 =$   
 $777 + 200 =$

What patterns do you notice?

$235 + 3 =$   
 $235 + 30 =$   
 $235 + 300 =$

$111 + \square = 118$   
 $604 + 20 =$   
 $604 + 50 =$   
 $604 + 90 =$

$111 + \square = 181$   
 $111 + \square = 811$

**Add two numbers (no exchange)**  
Mental strategies and introduction of formal written method.

... ones + ... ones = ... ones  
 ... tens + ... tens = ... tens  
 ... hundreds + ... hundreds = ... hundreds

$345 + 432 =$

**Add two numbers across a 10 or 100**  
Formal written method involving up to 2 exchanges including 3-digit plus 2-digit numbers.

There are ... ones, so I do/do not need to make an exchange.  
 There are ... tens, so I do/do not need to make an exchange.  
 ... ones = ... ten and ... ones.  
 ... tens = ... hundred and ... tens.

$466 + 353 =$

**Complements to 100**  
Pairs of numbers which total 100

... plus ... is equal to 100

$38 + 62 = 100$

I add ... to get to the next 10, then ... to get to 100

$38 + 62 = 100$   
 $62 + 38 = 100$   
 $100 = 38 + 62$   
 $100 = 62 + 38$

**Add fractions with the same denominator within 1 whole**  
 Make links with known facts.

When adding fractions with the same denominator, I only add the numerator.  
 ... fifths + ... fifths = ... fifths

$\frac{2}{5} + \frac{1}{5}$   
 $\frac{1}{5} + \frac{2}{5}$   
 $\frac{1}{5} + \frac{3}{5}$

**Calculate the duration of events**  
 Find durations of time between a given start and end point.  
 Children will need to calculate complements to 60

From ... to ... o'clock is ... minutes.  
 From ... o'clock to ... is ... minutes.  
 The total time taken is ... minutes.

start finish

4:25 4:55

start finish

+ 35 mins + 18 mins

2:25 3:00 3:18

**Year 4**

- Add numbers with up to 4 digits using a formal written method.
- Solve simple measure and money problems involving fractions and decimals to 2 decimal places.
- Add fractions with the same denominator.

**Progression of skills**      **Key representations**

**Add 1s, 10s and 100s to a 4-digit number**  
 Emphasis on mental strategies including number bonds and related facts. Prompt children to notice which digit changes.

The ones/tens/hundreds/thousands column will increase by ...

Thousands	Hundreds	Tens	Ones
1,000 1,000 1,000	100 100 100 100	10 10	1 1 1 1 1

$3,425 + 3 =$                        $3,425 + 30 =$   
 $3,425 + 300 =$                        $3,425 + 3,000 =$

What patterns do you notice?

$2,350 + 3 =$   
 $2,350 + 30 =$   
 $2,350 + 300 =$   
 $2,350 + 3,000 =$

$6,040 + 200 =$        $2,211 + \square = 2,251$   
 $6,040 + 500 =$        $2,211 + \square = 2,215$   
 $6,040 + 900 =$        $2,211 + \square = 2,511$

**Add up to two 4-digit numbers**  
 Formal written method with up to 3 exchanges. Encourage children to estimate and use inverse operations to check answers to calculations.

There are ... ones/tens/hundreds so I do/do not need to make an exchange.


I can exchange 10 ... for 1 ...

Th	H	T	O
1,000 1,000 1,000	100 100 100 100 100 100	10 10 10 10 10	1 1 1 1 1 1 1 1

Th	H	T	O
4	6	7	3
+	1	5	1
<hr/>			
6	1	9	1
1			

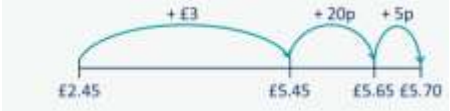
**Add decimal numbers in the context of money**  
 Emphasis on partitioning and use of number lines rather than formal written calculations.

... pence + ... pence = ... pence  
 ... pounds + ... pounds = ... pounds



$45p + 25p = 70p$   
 $£2 + £3 = £5$   
 $£5 + 70p = £5.70$

£3.25 can be partitioned into £3 + 20p + 5p



$£2.45 \xrightarrow{+£3} £5.45 \xrightarrow{+20p} £5.65 \xrightarrow{+5p} £5.70$

**Add fractions  
and mixed  
numbers with  
the same  
denominator  
beyond 1 whole**

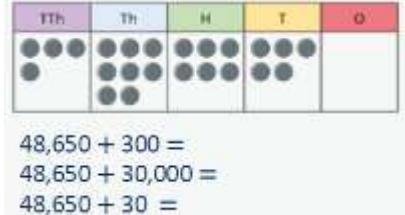
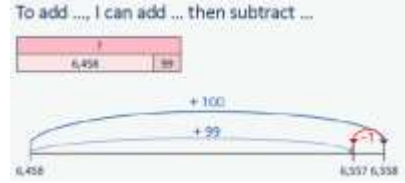
When adding fractions with the same denominator, I only add the numerator.  
... fifths + ... fifths = ... fifths

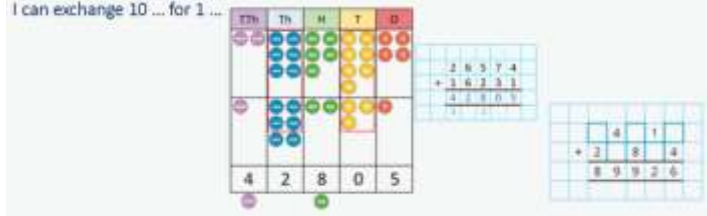
$\frac{3}{5} + \frac{4}{5} = \frac{7}{5} = 1 \frac{2}{5}$

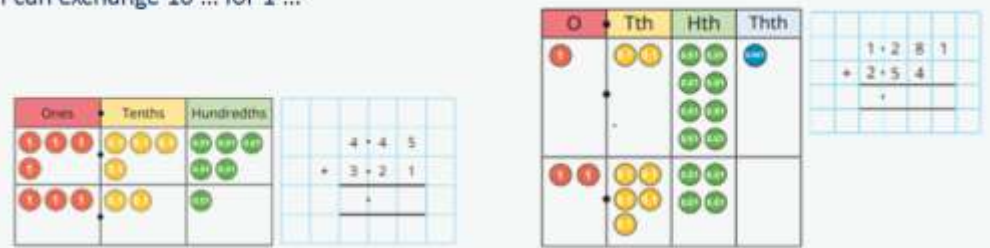
Diagram illustrating the addition of fractions with the same denominator. It shows a bar model with 10 blocks: 3 green, 4 yellow, and 3 white. A tree diagram shows 7/5 branching into 1 and 2/5. A number line from 0 to 3 shows a blue bar from 0 to 1 and a yellow bar from 1 to 1 2/5, with an arrow labeled 2/5 indicating the addition.

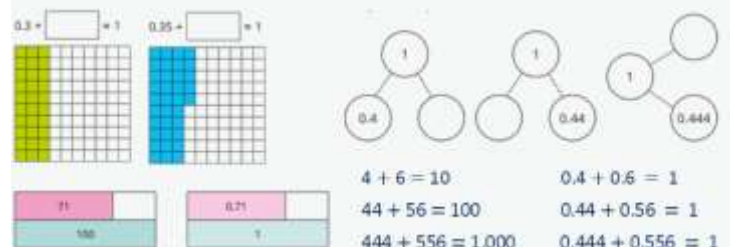
<b>Year 5</b>	<ul style="list-style-type: none"> <li>• Add whole numbers with more than 4 digits, including using formal written methods.</li> <li>• Add numbers mentally with increasingly large numbers.</li> <li>• Add decimals, including a mix of whole numbers and decimals, decimals with different numbers of decimal places, and complements of 1</li> <li>• Add fractions with the same denominator, and denominators that are multiples of the same number.</li> </ul>
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<b>Progression of skills</b>	<b>Key representations</b>
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<b>Add using mental strategies</b> Add 1s, 10s, 100s, etc. to any number. Use number bonds and related facts.		
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<b>Add whole numbers with more than 4 digits</b> Encourage children to estimate and use inverse operations to check answers to calculations.	
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<b>Add decimals with up to 2 decimal places</b> Progress from the same number of decimal places to a different number of decimal places, and from no exchange to exchange.	<p>I do/do not need to make an exchange because ...          I can exchange 10 ... for 1 ...</p> 
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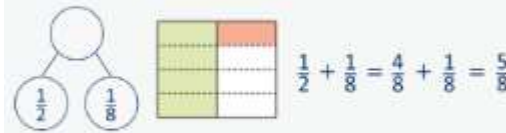
<b>Complements to 1</b> Pairs of numbers with up to 3 decimal places which total 1. Encourage children to make links with bonds to 10 and complements to 100 and 1,000.	
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**Add fractions with denominators that are a multiple of one another**

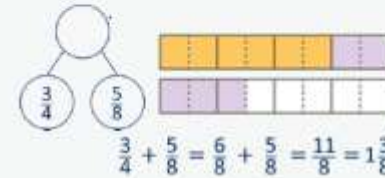
Encourage children to convert fractions to the same denominator before adding.

Progress from adding fractions within 1 whole to adding fractions beyond 1 whole.

The denominator has been multiplied by ..., so the numerator needs to be multiplied by... for the fractions to be equivalent.



$\frac{1}{4} + \frac{3}{8} = \frac{2}{8} + \frac{3}{8} = \frac{5}{8}$



**Year 6**

- Add larger numbers, using the formal written method of columnar addition.
- Use their knowledge of the order of operations to carry out calculations involving the 4 operations.
- Calculate intervals across zero.
- Add fractions with different denominators and mixed numbers, using the concept of equivalent fractions.

**Progression of skills**      **Key representations**

**Add integers up to 10 million**  
Encourage children to estimate and use inverse operations to check answers to calculations.

**Add decimals with up to 3 decimal places**  
Progress to numbers with digits in different place value columns. Encourage children to check that they have lined up the columns correctly.

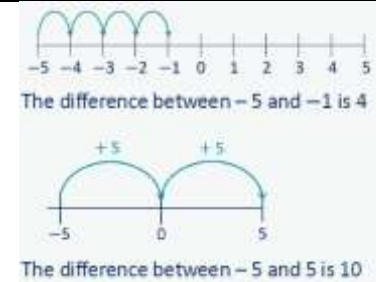
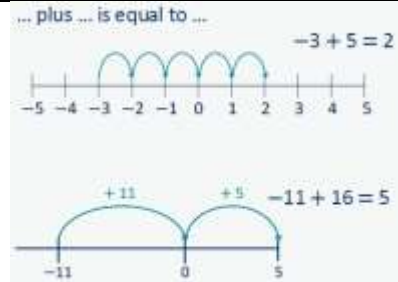
I do/do not need to make an exchange because ...

**Order of operations**  
Calculations in brackets should be done first. Multiplication and division should be performed before addition and subtraction. \*When no brackets are shown and the operations have the same priority, work left to right.

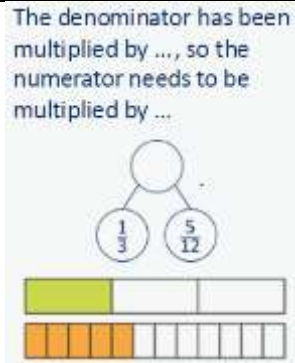
... has greater priority than ..., so the first part of the calculation I need to do is ...

**Negative numbers**

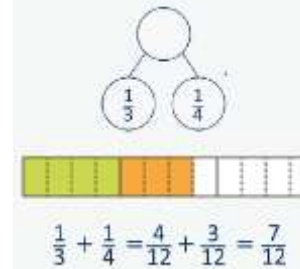
Children add to negative numbers and carry out calculations which cross 0

**Add fractions**

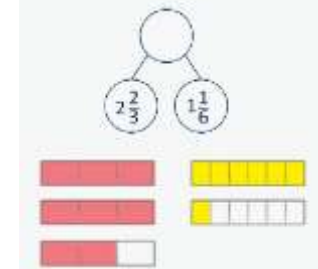
Convert fractions to the same denominator before adding. Progress from fractions where one denominator is a multiple of the other, to any fractions and then to mixed numbers.



The lowest common multiple of ... and ... is ...




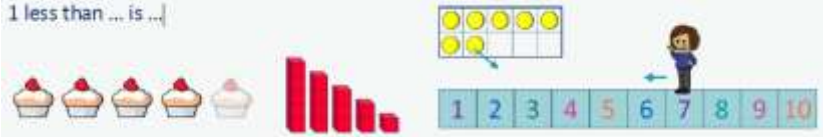






...is made up of ... wholes and ...

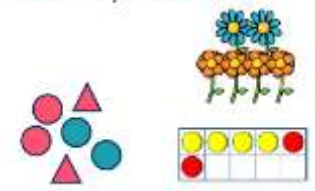
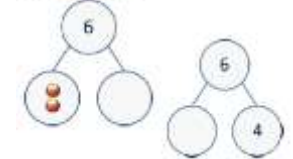
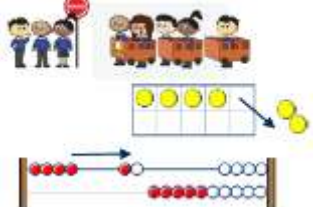
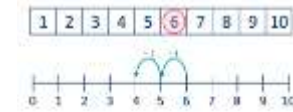
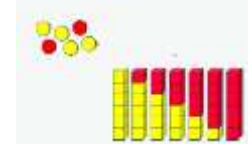
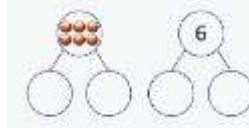
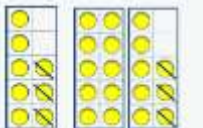
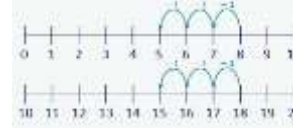


## Progression of skills – Subtraction

Year group	Skill
FS2	<ul style="list-style-type: none"> <li>• Conceptually subitise to 5</li> <li>• 1 less</li> <li>• Notice the composition of numbers within 10</li> <li>• Partition</li> <li>• Take away</li> </ul>
Year 1	<ul style="list-style-type: none"> <li>• Find a part</li> <li>• Take away</li> <li>• Bonds within 10</li> <li>• Related facts within 20</li> <li>• Missing numbers</li> </ul>
Year 2	<ul style="list-style-type: none"> <li>• Subtract 1s from any number (related facts)</li> <li>• Subtract across a 10</li> <li>• Subtract multiples of 10</li> <li>• Subtract 10s from any number</li> <li>• Subtract two 2-digit numbers (not across a ten)</li> <li>• Subtract two 2-digit numbers (across a ten)</li> <li>• Missing numbers</li> </ul>
Year 3	<ul style="list-style-type: none"> <li>• Subtract 1s, 10s and 100s from a 3-digit number</li> <li>• Subtract two numbers (no exchange)</li> <li>• Subtract two numbers across a 10 or 100</li> <li>• Complements to 100</li> <li>• Subtract fractions with the same denominator within 1 whole</li> </ul>
Year 4	<ul style="list-style-type: none"> <li>• Subtract 1s, 10s, 100s and 1,000s from a 4-digit number</li> <li>• Subtract up to two 4-digit numbers</li> <li>• Subtract decimal numbers in the context of money</li> <li>• Subtract fractions and mixed numbers with the same denominator</li> </ul>
Year 5	<ul style="list-style-type: none"> <li>• Subtract whole numbers with more than 4 digits</li> <li>• Subtract using mental strategies</li> <li>• Subtract decimals with up to 2 decimal places</li> <li>• Complements to 1</li> <li>• Subtract fractions with denominators that are a multiple of one another</li> </ul>
Year 6	<ul style="list-style-type: none"> <li>• Subtract integers up to 10 million</li> <li>• Subtract decimals with up to 3 decimal places</li> <li>• Order of operations</li> <li>• Negative numbers</li> <li>• Subtract fractions</li> </ul>

**Subtraction**

<p><b>FS2</b></p>	<ul style="list-style-type: none"> <li>• Have a deep understanding of number to 10, including the composition of each number.</li> <li>• Subitise (recognise quantities without counting) up to 5</li> <li>• Automatically recall (without reference to rhymes, counting or other aids) number bonds up to 5 (and some subtraction facts) and some number bonds to 10, including double facts.</li> </ul>	
<p><b>Progression of skills</b></p>	<p><b>Key representations</b></p>	
<p><b>Conceptually subitise to 5</b> Notice the parts that make up the whole.</p>	<p>What do you see? How do you see it?</p> 	
<p><b>1 less</b> Continue to link to stories, songs and rhymes.</p>	<p>1 less than ... is ...</p> 	
<p><b>Notice the composition of numbers within 10</b> Link to stories, songs and rhymes.</p>	<p>How many...? How many...? How many altogether?</p> 	<p>How many ways can you make...?</p> 
<p><b>Partition</b> Using objects, explore different ways to partition a number into 2 or more parts.</p>	<p>There are ... altogether. I can see ... here and ... there.</p> 	
<p><b>Take away</b> A quantity is reduced.</p>	<p>First... Then... Now...</p> 	<p>... and ... make ...</p>  <p>I have ... I take ... away Now I have ...</p> 

<b>Year 1</b>	<ul style="list-style-type: none"> <li>• Read, write and interpret mathematical statements involving subtraction (−) and equals (=) signs.</li> <li>• Represent and use number bonds and related subtraction facts within 20</li> <li>• Subtract one-digit and two-digit numbers to 20, including zero.</li> <li>• Solve one-step problems that involve subtraction, using concrete objects and pictorial representations, and missing number problems such as <math>7 = \square - 9</math></li> </ul>		
<b>Progression of skills</b>	<b>Key representations</b>		
<b>Find a part</b> Link to number bonds and known facts. E.g. $2 + 4 = 6$ so if 6 is the whole and 4 is a part, the other part must be 2	<p>There are ... in total.          ... are ...          How many are not ...?</p> 	<p>... is the whole.          ... is a part.          ... is a part.</p> 	<p>... subtract ... is equal to ...          ... is equal to ... − ...</p> $6 - 2 = 4$ $6 - 4 = 2$ $4 = 6 - 2$ $2 = 6 - 4$
<b>Take away</b> A quantity is decreased.	<p>First... Then... Now...</p> 	<p>I start at ...          I jump back ...          I land on ...</p> 	<p>... minus ... is equal to ...          ... is equal to ... − ...</p> $6 - 2 = 4$ $6 - 4 = 2$ $4 = 6 - 2$ $2 = 6 - 4$
<b>Bonds within 10</b> Focus on subtraction facts. Encourage children to notice patterns.	<p>... is made of ... and ...          ... and ... make ...</p> 	<p>... can be partitioned into ...          and ...</p> 	<p>... minus ... is equal to ...</p> $6 - 0 = 6$ $6 - 1 = 5$ $6 - 2 = 4$ $6 - 3 = 3$ $6 - 4 = 2$ $6 - 5 = 1$ $6 - 6 = 0$
<b>Related facts within 20</b> Make links to known facts.	<p>I know that ... minus ... = ...          so ... minus ... = ...</p> 	<p>... less than ... is ...          so ... less than ... is ...</p> 	<p>What patterns do you notice?</p> $8 - 3 = 5$ $18 - 3 = 15$ $5 = 8 - 3$ $15 = 18 - 3$

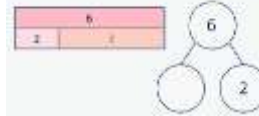
**Missing numbers**

Make links to known facts

How many do you need to subtract to make ...?



If ... is the whole and ... is a part, the other part must be...



... minus ... is equal to ...

$$6 - \square = 2$$
$$2 = 6 - \square$$



<b>Year 2</b>	<ul style="list-style-type: none"> <li>Recall and use subtraction facts to 20 fluently, and derive and use related facts up to 100             <ul style="list-style-type: none"> <li>Subtract numbers using concrete objects, pictorial representations, and mentally, including: a two-digit number and 1s</li> <li>a two-digit number and 10s</li> <li>2 two-digit numbers</li> </ul> </li> <li>Recognise and use the inverse relationship between addition and subtraction and use this to check calculations and solve missing number problems.</li> </ul>
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<b>Progression of skills</b>	<b>Key representations</b>
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**Subtract ones from any number**  
(related facts)  
Make links to known facts.

I know that ... minus ... = ...  
so ... minus ... = ...

... less than ... is ...  
so ... less than ... is ...

What do you notice?  
Can you continue the pattern?

$$8 - 3 = 5$$

$$18 - 3 = 15$$

$$28 - 3 = 25...$$

**Subtract across a 10**  
Partition the number being subtracted to bridge through a ten.

... can be partitioned into ... and ...

Make links with related facts.

**Subtract multiples of 10**  
Make links to known facts within ten.

... ones - ... ones = ... ones  
so ... tens - ... tens = ... tens

$5 - 2 = 3$   
 $50 - 20 = 30$

What is the same?  
What is different?

5	3
2	3

50	30
20	30

**Subtract 10s from any number**  
Make links to known facts.

... tens - ... tens = ... tens  
... tens and ... ones = ...

To subtract ... I need to subtract 10 ... times.

I know that ... minus ... = ...  
so ... minus ... = ...

$$50 - 20 = 30$$

$$54 - 20 = 34$$

**Subtract two 2-digit numbers**  
(not across a ten)

... ones - ... ones = ... ones  
... tens - ... tens = ... tens

3 ones - 1 one = 2 ones  
4 tens - 2 tens = 2 tens  
2 tens and 2 ones = 22

**Subtract two 2-digit numbers**  
(across a ten)  
Begin to exchange 1 ten for 10 ones.

I need to make an exchange because I do not have enough ones to subtract ... ones.

3 ones - 5 ones  
(I need to exchange 1 ten for 10 ones)

13 ones - 5 ones = 8 ones  
3 tens - 2 tens = 1 ten  
1 ten and 8 ones = 18

**Missing numbers**  
Solve missing number problems and use the inverse to check.

How many do you need to subtract to make ...?

$10 - \square = 6$   
 $6 + \square = 10$

If ... is a whole and ... is a part, then ... is the other part.

$7 - 3 = \square$   
 $\square + 3 = 7$

... can be partitioned into ... and ...

$18 - \square = 12 + 2$

**Year 3**

- Subtract numbers mentally, including: a three-digit number and ones, a three-digit number and tens, a three-digit number and hundreds.
- Subtract numbers with up to three digits, using formal written methods.
- Subtract fractions with the same denominator within 1 whole.

**Progression of skills**      **Key representations**

**Subtract 1s, 10s and 100s from a 3-digit number**  
 Emphasis on mental strategies including number bonds and related facts. Prompt children to notice which digit changes.

The ones/tens/hundreds column will decrease by ...

$444 - 2 =$                        $777 - 4 =$   
 $444 - 20 =$                      $777 - 40 =$   
 $444 - 200 =$                     $777 - 400 =$

What patterns do you notice?

$235 - 3 =$   
 $235 - 30 =$   
 $235 - 300 =$

$118 - \square = 111$   
 $624 - 20 =$        $181 - \square = 111$   
 $654 - 50 =$        $811 - \square = 111$   
 $694 - 90 =$

**Subtract two numbers (no exchange)**  
 Mental strategies and introduction of formal written method.

... ones - ... ones = ... ones  
 ... tens - ... tens = ... tens  
 ... hundreds - ... hundreds = ... hundreds

769	
147	?

**Subtract two numbers across a 10 or 100**  
 Formal written method involving up to 2 exchanges including 3-digit subtract 2-digit numbers.

I need to subtract ... ones. I do/do not need to make an exchange.  
 I need to subtract ... tens. I do/do not need to make an exchange.  
 I can exchange 1 ... for 10 ...

**Complements to 100**  
 Focus on subtraction facts. Encourage children to notice patterns.

100 minus ... is equal to ...

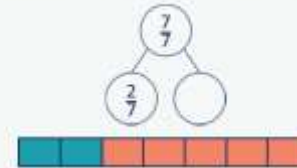
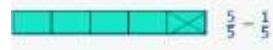
I subtract ... tens, then I subtract ... ones.

$100 - 38 = 62$   
 $100 - 62 = 38$   
 $62 = 100 - 38$   
 $38 = 100 - 62$

**Subtract fractions  
with the same  
denominator  
within 1 whole**  
Make links with  
known facts.

When subtracting fractions with the same denominator, I only subtract the numerator.

... fifths = ... fifths = ... fifths



**Year 4**

- Subtract numbers with up to 4 digits using a formal written method.
- Solve simple measure and money problems involving fractions and decimals to 2 decimal places.
- Subtract fractions with the same denominator.

**Progression of skills**      **Key representations**

**Subtract 1s, 10s, 100s and 1,000s from a 4-digit number**  
 Emphasis on mental strategies including number bonds and related facts. Prompt children to notice which digit changes.

The ones/tens/hundreds/thousands column will decrease by ...

Thousands	Hundreds	Tens	Ones
1,000 1,000 1,000	100 100 100 100	10 10	1 1 1 1 1

$3,425 - 2 =$        $3,425 - 200 =$   
 $3,425 - 20 =$      $3,425 - 2,000 =$

What patterns do you notice?

$4,356 - 3 =$   
 $4,356 - 30 =$   
 $4,356 - 300 =$   
 $4,356 - 3,000 =$

$4,433 - \square = 4,430$   
 $4,433 - \square = 4,033$   
 $4,433 - \square = 4,403$

$6,940 - 200 =$   
 $6,940 - 300 =$   
 $6,940 - 400 =$

**Subtract up to two 4-digit numbers**  
 Formal written method with up to 3 exchanges. Encourage children to estimate and use inverse operations to check answers to calculations.

I need to subtract... ones/tens/hundreds. I do/do not need to make an exchange.

I can exchange 1... for 10...

Th	H	T	O
3	4	2	6
			8
			0
			5
			8

Th	H	T	O
3	4	2	6
			8
			0
			5
			8

Subtract decimal numbers in the context of money  
 Emphasis here is on partitioning and use of number lines rather than formal written calculations.

I can partition £... into £... and 100p

$£... - £... = £...$   
 $100p - ...p = ...p$

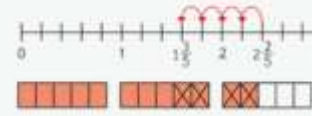
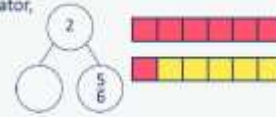
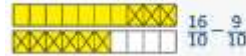
$£5 - £3.26$   
 $£4 - £3 = £1$   
 $100p - 26p = 74p$   
 $£5 - £3.26 = £1.74$

£3.26 can be partitioned into £3 + 20p + 6p

**Subtract fractions and mixed numbers with the same denominator**

Include subtracting fractions from wholes.

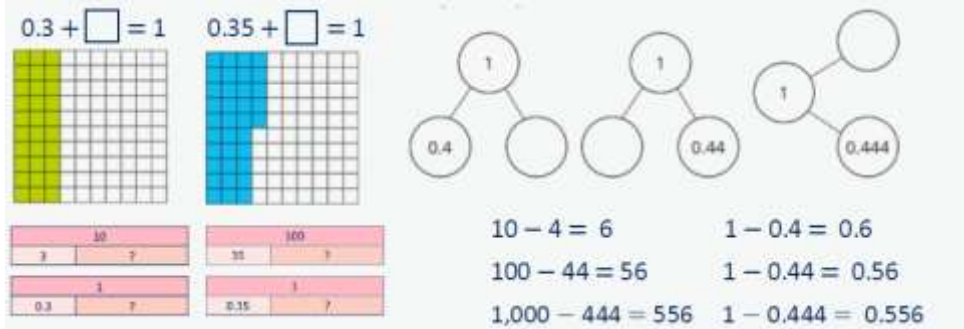
When subtracting fractions with the same denominator, I only subtract the numerator.  
... tenths - ... tenths = ... tenths





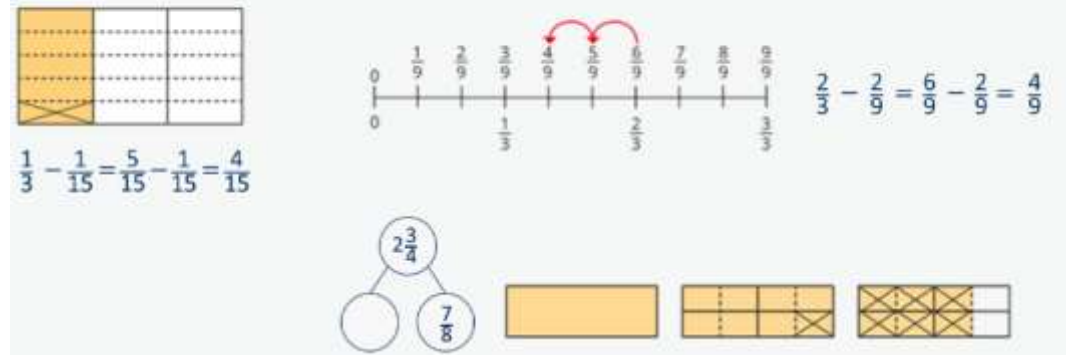
**Complements to 1**

Encourage children to make links with bonds to 10 and complements to 100 and 1,000 when finding a missing part or subtracting from 1

**Subtract fractions with denominators that are a multiple of one another**

Convert fractions to the same denominator before subtracting. Progress from subtracting fractions within 1 whole to subtracting from a mixed number.

The denominator has been multiplied by ..., so the numerator needs to be multiplied by... for the fractions to be equivalent.



**Year 6**

- Subtract larger numbers, using the formal written methods of columnar subtraction.
- Use their knowledge of the order of operations to carry out calculations involving the 4 operations.
- Calculate intervals across zero.
- Subtract fractions with different denominators and mixed numbers, using the concept of equivalent fractions.

**Progression of skills**      **Key representations**

**Subtract integers up to 10 million**  
Encourage children to estimate and use inverse operations to check answers to calculations.

**Subtract decimals with up to 3 decimal places**  
Progress from the same number of decimal and whole number places to a different number of decimal and whole number places

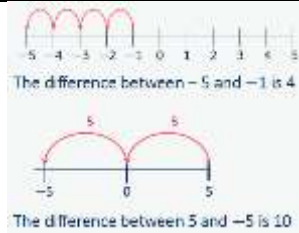
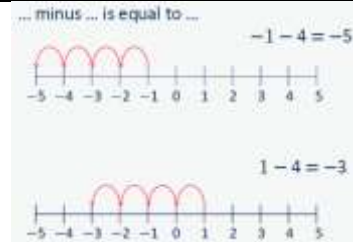
I do/do not need to make an exchange because ...

**Order of operations**  
Children learn the order of priority for operations in a calculation. Calculations in brackets should be done first. Multiplication and division should be performed before addition and subtraction.

... has greater priority than ..., so the first part of the calculation I need to do is ...

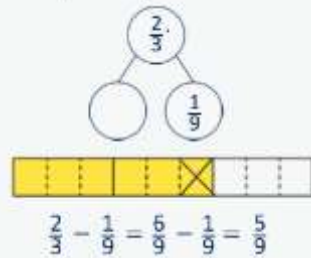
**Negative numbers**

Children subtract from positive and negative numbers and calculate intervals across 0

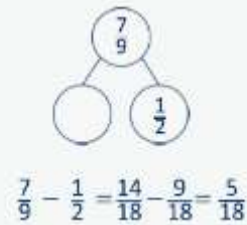
**Subtract fractions**

Convert fractions to the same denominator before subtracting. Progress from fractions where one denominator is a multiple of the other, to any fractions and then subtracting from a mixed number.

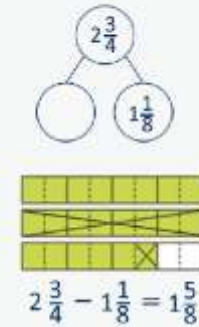
The denominator has been multiplied by ..., so the numerator needs to be multiplied by...



The lowest common multiple of ... and ... is ...



... is made up of ... wholes and ...



**Progression of skills – Multiplication**

Year group	Skill	
FS2	<ul style="list-style-type: none"> <li>• Double to 10</li> <li>• Make equal groups</li> </ul>	
Year 1	<ul style="list-style-type: none"> <li>• Count in 2s, 5s and 10s</li> <li>• Add equal groups</li> <li>• Make arrays</li> <li>• Make doubles</li> </ul>	
Year 2	<ul style="list-style-type: none"> <li>• Link repeated addition and multiplication</li> <li>• Use arrays</li> <li>• Double</li> <li>• The 2 times-table</li> <li>• The 10 times-table</li> <li>• The 5 times-table</li> <li>• Missing numbers</li> </ul>	
Year 3	<ul style="list-style-type: none"> <li>• The 3 times-table</li> <li>• The 4 times-table</li> <li>• The 8 times-table</li> <li>• Related facts</li> </ul>	<ul style="list-style-type: none"> <li>• Multiply a 2-digit number by a 1-digit number - no exchange</li> <li>• Multiply a 2-digit number by a 1-digit number - with exchange</li> <li>• Scaling</li> <li>• Correspondence problems</li> </ul>
Year 4	<ul style="list-style-type: none"> <li>• Times-table facts to <math>12 \times 12</math></li> <li>• Multiply by 1 and 0</li> <li>• Multiply 3 numbers</li> <li>• Factor pairs</li> <li>• Multiply by 10 and 100</li> </ul>	<ul style="list-style-type: none"> <li>• Related facts</li> <li>• Mental strategies</li> <li>• Multiply a 2 or 3-digit number by a 1-digit number</li> <li>• Scaling</li> <li>• Correspondence problems</li> </ul>
Year 5	<ul style="list-style-type: none"> <li>• Multiples and factors</li> <li>• Square and cube numbers</li> <li>• Multiply numbers up to 4 digits by a 1-digit number</li> <li>• Multiply numbers up to 4 digits by a 2-digit number</li> <li>• Multiply by 10, 100 and 1,000</li> </ul>	<ul style="list-style-type: none"> <li>• Mental strategies</li> <li>• Multiply fractions by a whole number</li> <li>• Multiply mixed numbers by a whole number</li> <li>• Find the whole</li> </ul>
Year 6	<ul style="list-style-type: none"> <li>• Multiply numbers up to 4 digits by a 2-digit number</li> <li>• Multiply by 10, 100 and 1,000</li> <li>• Order of operations</li> <li>• Multiply decimals by integers</li> <li>• Multiply fractions by fractions</li> <li>• Find the whole</li> <li>• Calculations involving ratio</li> </ul>	

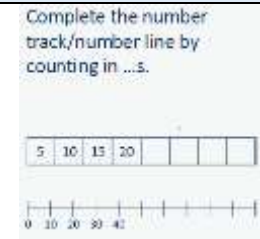


**Year 1**

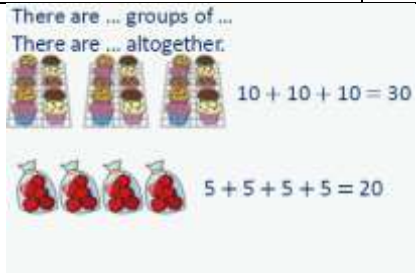
- Count in multiples of twos, fives and tens.
- Solve one-step problems involving multiplication, using concrete objects, pictorial representations and arrays with the support of the teacher

**Progression of skills**      **Key representations**

**Count in 2s, 5s and 10s**  
Begin by counting objects that naturally come in 2s, 5s and 10s, for example pairs of socks or fingers.



**Add equal groups (repeated addition)**  
Children should be able to write a repeated addition to represent equal groups and to draw pictures or use objects to represent a repeated addition.



What is the same? What is different?

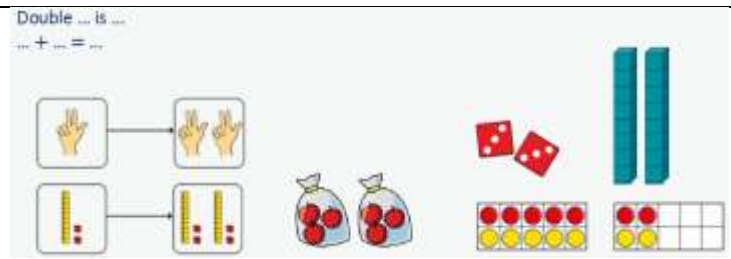
$2 + 2 + 2 =$   
 $5 + 5 + 5 =$   
 $10 + 10 + 10 =$

Use objects or a drawing to represent the equal groups and find how many in total.

**Make arrays**  
Children use their knowledge of adding equal groups to arrange objects in columns and rows.

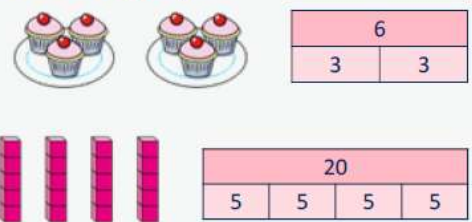



**Make doubles**  
Children understand that doubles are two equal groups.  
Children may begin to explore doubles beyond 20 using base 10





<b>Year 2</b>	<ul style="list-style-type: none"> <li>Recall and use multiplication facts for the 2, 5 and 10 multiplication tables.</li> <li>Calculate mathematical statements for multiplication within the multiplication tables and write them using the multiplication (×) and equals (=) signs.</li> <li>Show that multiplication of two numbers can be done in any order (commutative).</li> </ul>
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<b>Progression of skills</b>	<b>Key representations</b>
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<p><b>Link repeated addition and multiplication</b></p> <p>Encourage children to make the link between repeated addition and multiplication.</p>	<p>There are ... equal groups with ... in each group. There are ... altogether.</p>  <p> <math>3 + 3 = 6</math>  <math>2 \times 3 = 6</math> </p> <p> <math>5 + 5 + 5 + 5 = 20</math>  <math>4 \times 5 = 20</math> </p>
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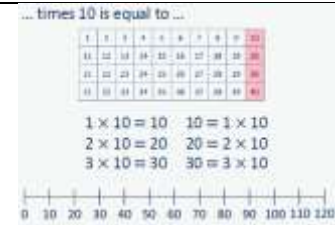
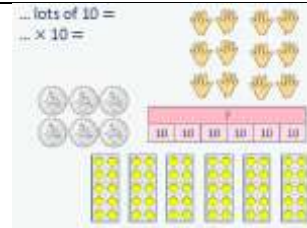
<p><b>Use arrays</b></p> <p>Encourage children to see that multiplication is commutative.</p>	<p>There are ... rows with ... in each row. There are ... columns with ... in each column.</p>  <p>         3 lots of 5 = 15  <math>5 + 5 + 5 = 15</math>          5 lots of 3 = 15  <math>3 + 3 + 3 + 3 + 3 = 15</math> </p>	<p>I can see ... × ... and ... × ...</p> <p> <math>3 \times 5 = 15</math>  <math>5 \times 3 = 15</math>  <math>3 \times 5 = 5 \times 3</math> </p>
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<p><b>Double</b></p> <p>Encourage children to make links with related facts.</p>	<p>Double ... is ...</p>  <p>         Double 4 = 4 + 4          Double 4 is 8       </p>	<p>Double ... is ... so double ... is ...</p>  <p>         Double 4 is 8          Double 40 is 80       </p>
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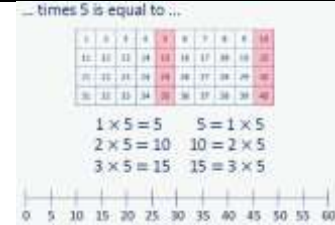
<p><b>The 2 times-table</b></p> <p>Encourage daily counting in multiples both forwards and back. Notice that all multiples of 2 are even numbers.</p>	<p>... lots of 2 = ... × 2 =</p> 	<p>... times 2 is equal to ...</p>  <p> <math>1 \times 2 = 2</math>   <math>2 = 1 \times 2</math>  <math>2 \times 2 = 4</math>   <math>4 = 2 \times 2</math>  <math>3 \times 2 = 6</math>   <math>6 = 3 \times 2</math> </p>
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**The 10 times-table**

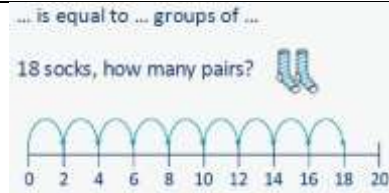
Encourage daily counting in multiples both forwards and back. Notice the pattern in the numbers.

**The 5 times-table**

Encourage daily counting in multiples both forwards and back. Notice the pattern in the numbers.

**Missing numbers**

Make links to known facts.



... times ... is equal to ...

$$\square \times 2 = 18$$

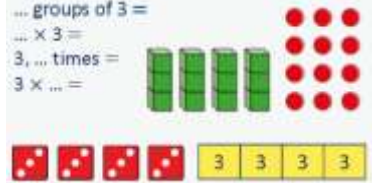
$$18 = 2 \times \square$$

<b>Year 3</b>	<ul style="list-style-type: none"> <li>Recall and use multiplication facts for the 3, 4 and 8 multiplication tables.</li> <li>Write and calculate mathematical statements for multiplication using the multiplication tables that they know, including for two-digit numbers times one-digit numbers, using mental and progressing to formal written methods.</li> <li>Solve problems, including missing number problems, involving multiplication, including positive integer scaling problems and correspondence problems in which n objects are connected to m objects.</li> </ul>
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<b>Progression of skills</b>	<b>Key representations</b>
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**The 3 times-table**  
Encourage daily counting in multiples both forwards and back.

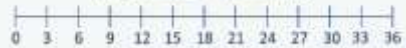
... groups of 3 =  
...  $\times 3 =$   
3, ... times =  
 $3 \times \dots =$



... times 3 is equal to ...

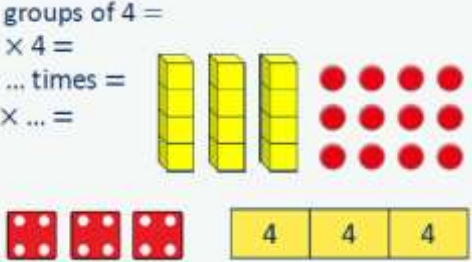
1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30

$4 \times 3 = 12$     $12 = 4 \times 3$



**The 4 times-table**  
Encourage daily counting in multiples both forwards and back. Encourage children to notice links between the 2 and 4 times-tables.

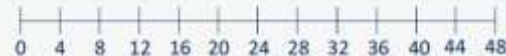
... groups of 4 =  
...  $\times 4 =$   
4, ... times =  
 $4 \times \dots =$



... times 4 is equal to ...

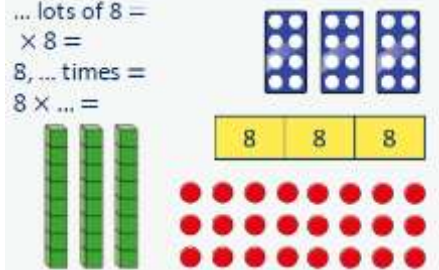
1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30

$3 \times 4 = 12$     $12 = 3 \times 4$



**The 8 times-table**  
Encourage daily counting in multiples both forwards and back. Encourage children to notice links between the 2, 4 and 8 times-tables.


... lots of 8 =  
 $\times 8 =$   
8, ... times =  
 $8 \times \dots =$



... times 8 is equal to ...

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30

$3 \times 8 = 24$     $24 = 3 \times 8$



**Related facts**

Use knowledge of multiplying by 10 to scale times-table facts.

... X ... ones is equal to ... ones  
so ... X ... tens is equal to ... tens.

$3 \times 4 = 12$   
 $3 \times 40 = 120$

**Multiply a 2-digit number by a 1-digit number - no exchange**

Children apply their understanding of partitioning to represent and solve calculations using the expanded method.

... tens multiplied by ... is equal to ... tens.  
...ones multiplied by ... is equal to ... ones.

Tens	Ones

$30 \times 2 = 60$   
 $2 \times 2 = 4$

$32 \times 2 = 64$

$21 \times 4$

$20 \times 4$     $1 \times 4$

Tens	Ones

**Multiply a 2-digit number by a 1-digit number - with exchange**

Children apply their understanding of partitioning to represent and solve calculations using the expanded method.

... tens multiplied by ... is equal to ... tens.  
... ones multiplied by ... is equal to ... ones.

Tens	Ones

$20 \times 4 = 80$   
 $4 \times 4 = 16$

$24 \times 4 = 96$

$45 \times 3$


$40 \times 3$     $5 \times 3$

Tens	Ones

### Scaling


Children focus on multiplication as scaling ( ... times the size) as opposed to repeated addition.

There are ... times as many ... as ...



There are 3 times as many triangles as circles.

... is ... times the size of ...  
... is ... times the length/height of ...



Miss Smith is twice the height of Jo.

### Correspondence problems

(How many ways?)  
Encourage children to work systematically to find all the different possible combinations.

For every ... , there are ... possible ...  
There are ...  $\times$  ... possibilities altogether.



	hats	scarves
blue		
orange		
purple		

For every hat, there are two possible scarves.  
 $3 \times 2 = 6$

There are 6 possibilities altogether.

- Year 4**
- Recall multiplication facts for multiplication tables up to  $12 \times 12$
  - Use place value, known and derived facts to multiply mentally, including: multiplying by 0 and 1; multiplying together three numbers.
  - Recognise and use factor pairs and commutativity in mental calculations.
  - Multiply two-digit and three-digit numbers by a one-digit number using formal written layout.
  - Solve problems involving multiplying and adding, including using the distributive law to multiply two-digit numbers by one digit, integer scaling problems and harder correspondence problems such as n objects are connected to m objects.

**Progression of skills**      **Key representations**

**Times-table facts to  $12 \times 12$**   
Encourage daily counting in multiples both forwards and back. Encourage children to notice links between related times-tables.

... groups of ... =  
... times ... is equal to ...  
...  $\times$  ... =

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

**Multiply by 1 and 0**

Any number multiplied by 1 is equal to ...  
Any number multiplied by 0 is equal to ...

...  $\times$  ... = ...

$1 \times 1 = 1$	$1 \times 0 = 0$
$2 \times 1 = 2$	$2 \times 0 = 0$
$3 \times 1 = 3$	$3 \times 0 = 0$
$4 \times 1 = 4$	$4 \times 0 = 0$

**Multiply 3 numbers**  
Children use their understanding of commutativity to multiply more efficiently.

To work out ...  $\times$  ...  $\times$  ..., I can first calculate ...  $\times$  ... and then multiply the answer by ...

$4 \times 2 \times 3 = 8 \times 3 = 24$   
 $2 \times 3 \times 4 = 6 \times 4 = 24$   
 $3 \times 4 \times 2 = 12 \times 2 = 24$

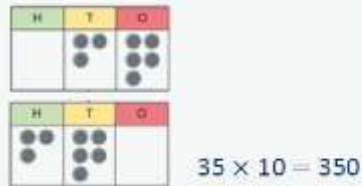
**Factor pairs**

Children explore equivalent calculations using different factors pairs.

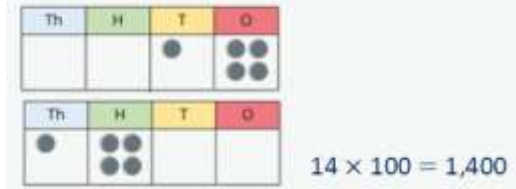
**Multiply by 10 and 100**

Some children may over-generalise that multiplying by 10 or 100 always results in adding zeros. This will cause issues later when adding decimals.

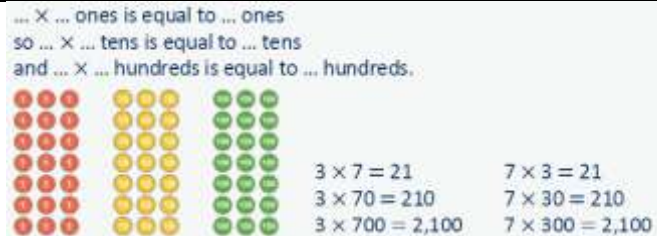
When I multiply by 10, the digits move ...  
place value column to the left.  
... is 10 times the size of ...



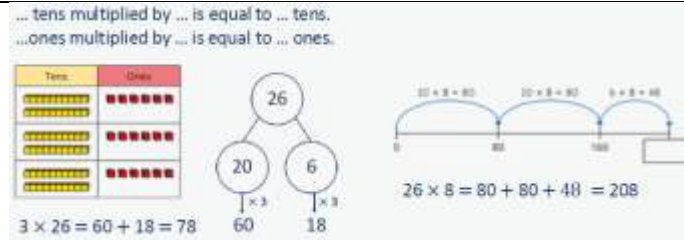
When I multiply by 100, the digits move ...  
place value columns to the left.  
... is 100 times the size of ...

**Related facts**

Use knowledge of multiplying by 10 and 100 to scale times-table facts.

**Mental strategies**

Partition 2 or 3-digit numbers to multiply using informal methods.



**Multiply a 2 or 3-digit number by a 1-digit number**

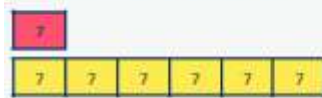
The short multiplication method is introduced for the first time, initially in an expanded form.

To multiply a 2-digit number by ..., I multiply the ones by ... and the tens by ...  
 To multiply a 3-digit number by ..., I multiply the ones by ..., the tens by ... and the hundreds by ...

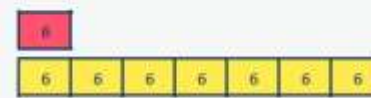
**Scaling**

Children focus on multiplication as scaling ( ... times the size).

... is ... times the size of ...



A computer mouse costs £7  
 A keyboard costs 6 times as much.



A red ribbon is 6 cm.  
 A yellow ribbon is 7 times as long.

**Correspondence problems**



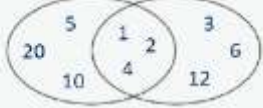


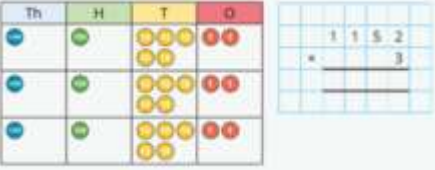
Encourage children to use tables to show all the different possible combinations.

For every ... , there are ... possibilities.  
 There are ... × ... possibilities altogether.

A pizza company offers a choice of 5 toppings and 3 bases.

$5 \times 3 = 15$

	Deep pan	Italian	Thin
Cheese	C DP	C I	C Th
Mushroom	M DP	M I	M Th
Vegetable	V DP	V I	V Th
Chicken	C DP	C I	C Th
Tuna	T DP	T I	T Th

<p><b>Year 5</b></p>	<ul style="list-style-type: none"> <li>Identify multiples and factors, including finding all factor pairs of a number, and common factors of two numbers</li> <li>Recognise and use square numbers and cube numbers, and the notation for squared (<math>^2</math>) and cubed (<math>^3</math>)</li> <li>Multiply numbers up to 4 digits by a one- or two-digit number using a formal written method, including long multiplication for two-digit numbers.</li> <li>Multiply numbers mentally drawing upon known facts.</li> <li>Multiply whole numbers and those involving decimals by 10, 100 and 1000</li> <li>Multiply proper fractions and mixed numbers by whole numbers, supported by materials and diagrams.</li> </ul>		
<p><b>Progression of skills</b></p>	<p><b>Key representations</b></p>		
<p><b>Multiples and factors</b> Encourage children to notice patterns and make links with known facts.</p>	<p>... is a multiple of ... because ... <math>\times</math> ... = ...</p> 	<p>... is a factor of ... because ... <math>\times</math> ... = ...</p>  <p>1, 2, 4 and 8 are factors of 8</p>	<p>The common factors of ... and ... are ...</p> <p>Factors of 20      Factors of 12</p> 
<p><b>Square and cube numbers</b></p>	<p>... squared means ... <math>\times</math> ...</p>  <p> <math>1 \times 1</math>    <math>2 \times 2</math>    <math>3 \times 3</math>    <math>4 \times 4</math>  <math>1^2 = 1</math>   <math>2^2 = 4</math>   <math>3^2 = 9</math>   <math>4^2 = 16</math> </p>	<p>... cubed means ... <math>\times</math> ... <math>\times</math> ...</p>  <p> <math>1 \times 1 \times 1</math>    <math>2 \times 2 \times 2</math>    <math>3 \times 3 \times 3</math>  <math>1^3 = 1</math>        <math>2^3 = 8</math>            <math>3^3 = 27</math> </p>	
<p><b>Multiply numbers up to 4 digits by a 1-digit number</b> This builds on the short multiplication method introduced in Y4</p>	<p>To multiply a 4-digit number by ..., I multiply the ones by ..., the tens by ..., the hundreds by ... and the thousands by ...</p> 		

**Multiply numbers up to 4 digits by a 2-digit number**

Numbers are first partitioned using an area model then long multiplication is introduced for the first time.

I can partition ... into ... and ...

x	40	4
30	1200	120
2	80	8

$32 \times 44 = 1,200 + 80 + 120 + 8$   
 $32 \times 44 = 1,408$

First, I multiply by the ... Then I multiply by the ...

$32 \times 4 = 128$   
 $32 \times 40 = 1280$   
 $32 \times 44 = 1408$

**Multiply by 10, 100 and 1,000**

Some children may over-generalise that multiplying by a power of 10 always results in adding zeros. This will cause issues later when multiplying decimals.

To multiply by 10/100/1,000, I move all the digits ... places to the left.  
 ... is 10/100/1,000 times the size of ...

M	HTh	TTh	Th	H	T	O
				●●	●●	●●

$234 \times 10 = 2,340$   
 $234 \times 100 = 23,400$   
 $234 \times 1,000 = 234,000$

Th	H	T	O	Tth	Hth
			●●	●●	●●

$2.34 \times 10 = 23.4$   
 $2.34 \times 100 = 234$   
 $2.34 \times 1,000 = 2,340$

**Mental strategies**

Children continue to use efficient mental strategies such as partitioning and knowledge of factor pairs and related facts to multiply.

The most efficient strategy to calculate ...  $\times$  ... is ...  
 To calculate ...  $\times$  12, I can do ...  $\times$  ...  $\times$  ...  
 For example:  $121 \times 12$   
 I could calculate  $100 \times 12$  plus  $20 \times 12$  plus  $1 \times 12$   
 I could calculate  $121 \times 10$  plus  $121 \times 2$   
 I could calculate  $121 \times 6 \times 2$   
 I could calculate  $121 \times 4 \times 3$

**Multiply fractions by a whole number**

Make links with repeated addition.

E.g.  $\frac{1}{5} \times 4 = \frac{1}{5} + \frac{1}{5} + \frac{1}{5} + \frac{1}{5}$

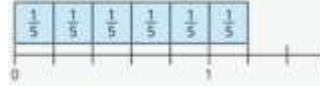
To multiply a fraction by an integer, I multiply the numerator by the integer and the denominator remains the same.



$$\frac{1}{7} \times 5 = \frac{1}{7} + \frac{1}{7} + \frac{1}{7} + \frac{1}{7} + \frac{1}{7} = \frac{5}{7}$$



$$\frac{2}{7} \times 3 = \frac{2}{7} + \frac{2}{7} + \frac{2}{7} = \frac{6}{7}$$



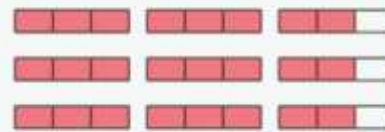
$$\frac{1}{5} \times 5 = \frac{5}{5} = 1$$



$$\frac{2}{5} \times 3 = \frac{6}{5} = 1\frac{1}{5}$$

**Multiply mixed numbers by a whole number**

I can partition  $\frac{2}{3}$  into  $\frac{2}{3}$  and  $\frac{2}{3}$



$$2\frac{2}{3} \times 3$$

$$2 \times 3 = 6 \quad \frac{2}{3} \times 3 = \frac{6}{3} = 2$$

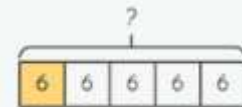
$$2\frac{2}{3} \times 3 = 6 + 2 = 8$$

**Find the whole**

Children multiply to find the whole from a given part.

If  $\frac{1}{5}$  is ... , then the whole is ...  $\times$  ...

$$\frac{1}{5} \text{ of } \underline{\quad} = 6$$

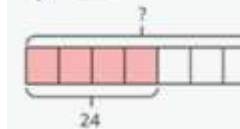


$$5 \times 6 = 30$$

$$\frac{1}{5} \text{ of } 30 = 6$$

If  $\frac{4}{7}$  is ... , then  $\frac{1}{7}$  is ... and the whole is ...  $\times$  ...

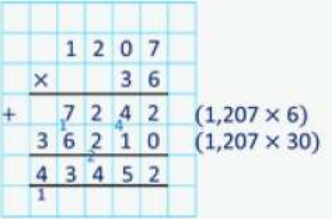
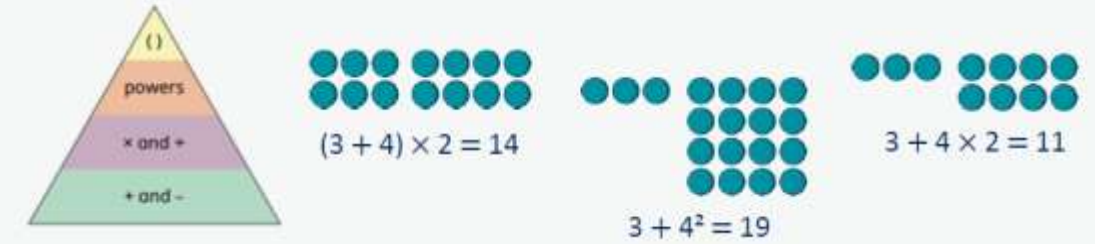
$$\frac{4}{7} \text{ of } \underline{\quad} = 24$$



$$\frac{1}{7} = 24 \div 4 = 6$$

$$7 \times 6 = 42$$

$$\frac{4}{7} \text{ of } 42 = 24$$

<p><b>Year 6</b></p>	<ul style="list-style-type: none"> <li>• Identify common factors and common multiples.</li> <li>• Multiply multi-digit numbers up to 4 digits by a two-digit whole number using the formal written method of long multiplication.</li> <li>• Multiply numbers by 10, 100 and 1,000</li> <li>• Multiply one-digit numbers with up to two decimal places by whole numbers.</li> <li>• Use their knowledge of the order of operations to carry out calculations involving the 4 operations.</li> <li>• Multiply simple pairs of proper fractions, writing the answer in its simplest form.</li> <li>• Solve problems involving the relative sizes of two quantities where missing values can be found by using integer multiplication and division facts</li> <li>• Solve problems involving the calculation of percentages.</li> </ul>																												
<p><b>Progression of skills</b></p>	<p><b>Key representations</b></p>																												
<p><b>Multiply numbers up to 4 digits by a 2-digit number</b></p>	<p>To multiply by a 2-digit number, first multiply by the ones, then multiply by the tens and then find the total.</p> 																												
<p><b>Multiply by 10, 100 and 1,000</b> Some children may over-generalise that multiplying by a power of 10 always results in adding zeros.</p>	<p>To multiply by 10/100/1,000, I move all the digits ... places to the left. ... is 10/100/1,000 times the size of ...</p> <table border="1" data-bbox="719 818 1765 917"> <tr> <td>M</td><td>HTh</td><td>TTh</td><td>Th</td><td>H</td><td>T</td><td>O</td><td>Th</td><td>H</td><td>T</td><td>O</td><td>Tth</td><td>Hth</td><td>Thth</td> </tr> <tr> <td></td><td></td><td></td><td></td><td>●●</td><td>●●</td><td>●●</td><td></td><td></td><td></td><td>●</td><td>●●</td><td>●●</td><td>●●</td> </tr> </table> <p> <math>234 \times 10 = 2,340</math>  <math>234 \times 100 = 23,400</math>  <math>234 \times 1,000 = 234,000</math> </p> <p> <math>0.234 \times 10 = 2.34</math>  <math>0.234 \times 100 = 23.4</math>  <math>0.234 \times 1,000 = 234</math> </p>	M	HTh	TTh	Th	H	T	O	Th	H	T	O	Tth	Hth	Thth					●●	●●	●●				●	●●	●●	●●
M	HTh	TTh	Th	H	T	O	Th	H	T	O	Tth	Hth	Thth																
				●●	●●	●●				●	●●	●●	●●																
<p><b>Order of operations</b> Calculations in brackets should be done first. Multiplication and division should be performed before addition and subtraction.</p>	<p>... has greater priority than ..., so the first part of the calculation I need to do is ...</p> 																												

**Multiply decimals by integers**

This is the first time children multiply decimals by numbers other than 10, 100 or 1,000

Encourage them to make links with known facts and whole number multiplication.

I know that ...  $\times$  ... = ..., so I also know that ...  $\times$  ... = ...

$6 \times 2 = 12$        $6 \times 0.2 = 1.2$

I need to exchange 10 ... for 1 ...

$213 \times 4 = 852$        $2.13 \times 4 = 8.52$

**Multiply fractions by fractions**

Encourage children to give answers in their simplest form.

When multiplying a pair of fractions, I need to multiply the numerator and multiply the denominator.

$\frac{1}{3} \times \frac{1}{5} = \frac{1}{15}$        $\frac{2}{3} \times \frac{4}{5} = \frac{8}{15}$        $\frac{2}{3} \times \frac{3}{5} = \frac{6}{15} = \frac{2}{5}$

**Find the whole**

Children multiply to find the whole from a given part.

If  $\frac{1}{3}$  is ..., then the whole is ...  $\times$  ...

$\frac{1}{3}$  of ... = 18

$18 \times 3 = 54$

$\frac{1}{3}$  of 54 = 18

If  $\frac{4}{9}$  is ..., then  $\frac{1}{9}$  is ... and the whole is ...  $\times$  ...

$\frac{4}{9}$  of ... = 48

$\frac{1}{9} = 48 \div 4 = 12$

$9 \times 12 = 108$

$\frac{4}{9}$  of 108 = 48

**Calculate percentages**

Children first learn how to find 1%, 10%, 20%, 25% and 50% before using multiples of these amounts to find any percentage.

There are ... lots of ... % in 100%

To find ... %, I need to divide by ...

100%			
50%		50%	
25%	25%	25%	25%

50% of ... = ...  $\div$  2

25% of ... = ...  $\div$  4

... % is made up of ... %, and ... %

100%									
10%	10%	10%	10%	10%	10%	10%	10%	10%	10%

To find 30%, I can find 10% and then multiply it by 3

To find 23%, I can use 10%  $\times$  2 and 1%  $\times$  3

To find 99%, I can find 1%, then subtract from 100%

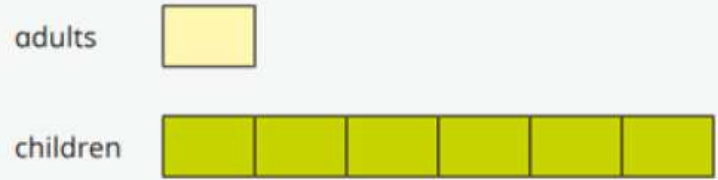
**Calculations involving ratio**

Encourage children to see the multiplicative relationship between ratios. They will need to multiply or divide each value by the same number to keep the ratio equivalent.

Double number lines and ratio tables help children to see both horizontal and vertical multiplicative relationships.

For every ... , there are ...

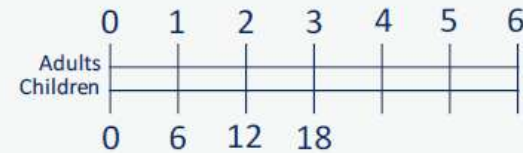
For every 1 adult on a school trip, there are 6 children.



Adults	Children
1	6
2	12
3	18

Diagram showing a ratio table with arrows indicating multiplication:  $\times 6$  (horizontal),  $\times 3$  (vertical), and  $\times 6$  (diagonal).

The ratio of adults to children is 1 : 6



**Progression of skills – Division**

Year group	Skill	
FS2	<ul style="list-style-type: none"> <li>• Sharing</li> <li>• Grouping</li> </ul>	
Year 1	<ul style="list-style-type: none"> <li>• Make equal groups – grouping</li> <li>• Make equal groups – sharing</li> <li>• Find a half</li> <li>• Find a quarter</li> </ul>	
Year 2	<ul style="list-style-type: none"> <li>• Divide by 2</li> <li>• Divide by 10</li> <li>• Divide by 5</li> <li>• Missing numbers</li> <li>• Unit fractions</li> <li>• Non-unit fractions</li> </ul>	
Year 3	<ul style="list-style-type: none"> <li>• Divide by 3</li> <li>• Divide by 4</li> <li>• Divide by 8</li> <li>• Related facts</li> <li>• Divide a 2-digit number by a 1-digit number - no exchange</li> </ul>	<ul style="list-style-type: none"> <li>• Divide a 2-digit number by a 1-digit number - with remainders</li> <li>• Unit fractions of a set of objects</li> <li>• Non-unit fractions of a set of objects</li> </ul>
Year 4	<ul style="list-style-type: none"> <li>• Division facts to <math>12 \times 12</math></li> <li>• Divide a number by 1 and itself</li> <li>• Related facts</li> </ul>	<ul style="list-style-type: none"> <li>• Divide a 2 or 3-digit number by a 1-digit number</li> <li>• Divide by 10 and 100</li> </ul>
Year 5	<ul style="list-style-type: none"> <li>• Mental strategies</li> <li>• Divide numbers up to 4 digits by a 1-digit number</li> <li>• Divide by 10, 100 and 1,000</li> <li>• Fraction of an amount</li> </ul>	
Year 6	<ul style="list-style-type: none"> <li>• Short division</li> <li>• Mental strategies</li> <li>• Long division</li> <li>• Order of operations</li> <li>• Divide by 10, 100 and 1,000</li> <li>• Divide decimals by integers</li> </ul>	<ul style="list-style-type: none"> <li>• Decimal and fraction equivalents</li> <li>• Divide a fraction by an integer</li> <li>• Fraction of an amount</li> <li>• Calculate percentages</li> <li>• Calculations involving ratio</li> </ul>

<b>FS2</b>	<ul style="list-style-type: none"> <li>• Have a deep understanding of number to 10, including the composition of each number.</li> <li>• Subitise (recognise quantities without counting) up to 5</li> <li>• Automatically recall (without reference to rhymes, counting or other aids) number bonds up to 5 and some number bonds to 10, including double facts.</li> <li>• Explore and represent patterns within numbers up to 10, including evens and odds, double facts and how quantities can be distributed equally.</li> </ul>
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<b>Progression of skills</b>	<b>Key representations</b>
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**Sharing**  
Provide practical activities such as sharing items during snack time. Encourage children to check whether items have been shared fairly (equally).

There are ... altogether.  
They are shared equally between ... groups.



**Grouping**  
Provide opportunities to make equal groups when tidying up or during snack time. Encourage children to check that each group has the same amount.

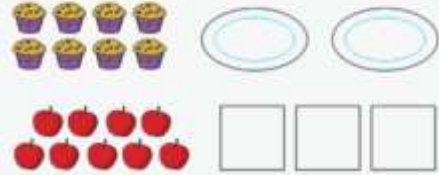

There are ... groups of ...  
There are ... altogether.


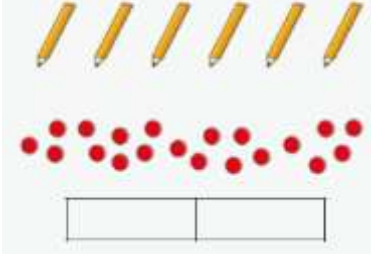
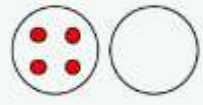


<p><b>Year 1</b></p>	<ul style="list-style-type: none"> <li>• Solve simple one-step problems involving division, using concrete objects, pictorial representations and arrays with the support of the teacher.</li> <li>• Recognise, find and name a half as one of two equal parts of a quantity.</li> <li>• Recognise, find and name a quarter as one of four equal parts of an object, shape or quantity.</li> </ul>
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<b>Progression of skills</b>	<b>Key representations</b>
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<p><b>Make equal groups - grouping</b> Encourage children to physically move objects into equal groups. They can also circle equal groups when using pictures.</p>	<p>There are ... altogether. How many groups of ... can you make?</p> 	<p>Circle groups of 2 There are ... groups of 2</p> 	<p>Take ... cubes. Make equal groups.</p>  <p>There are ... groups of ...</p>
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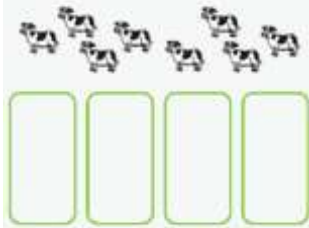
<p><b>Make equal groups - sharing</b> Encourage children to check that the objects have been shared fairly and each group is the same.</p>	<p>... have been shared equally between... There are ... on/in each ...</p> 	<p>Take ... cubes. Share them between ...</p>  <p>12 shared between ... is ...</p>
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<p><b>Find a half</b> Start with practical opportunities to share a quantity into 2 groups. Progress to circling half of the objects in a picture and then to finding the whole from a given half.</p>	<p>To find half, I need to share into 2 equal groups.</p>  <p>There are ... in each group.</p>	<p>Half of ... is ...</p> 	<p>If ... is half, what is the whole?</p>  <p>4 is half of ...</p>
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**Find a quarter**

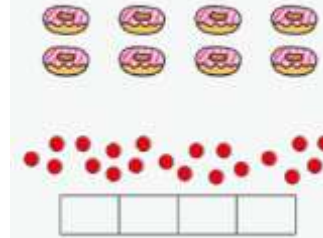
Start with practical opportunities to share a quantity into 4 groups. Progress to using pictures or bar models to find a quarter and then to finding the whole from a given quarter.

To find a quarter, I need to share into 4 equal groups.

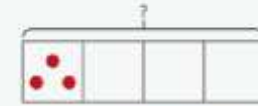


There are ... in each group.

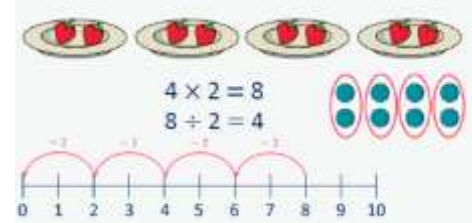
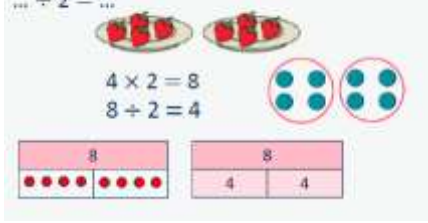
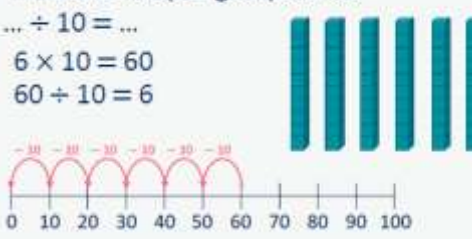
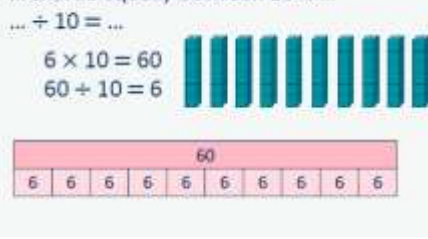
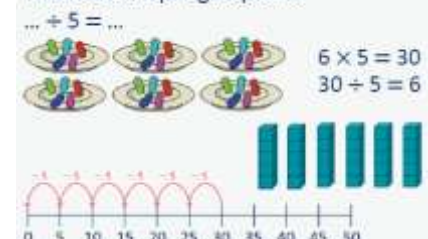
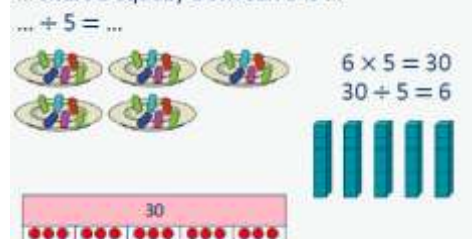
A quarter of ... is ...



If ... is one quarter, what is the whole?



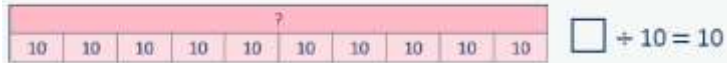
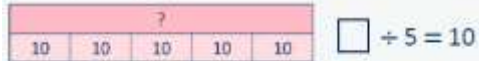
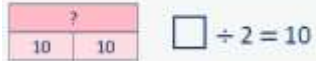
3 is one quarter of ...

<p><b>Year 2</b></p>	<ul style="list-style-type: none"> <li>Recall and use division facts for the 2, 5 and 10 multiplication tables.</li> <li>Calculate mathematical statements for division within the multiplication tables and write them using the division (<math>\div</math>) and equals (=) signs.</li> <li>Recognise, find, name and write fractions <math>\frac{1}{3}</math>, <math>\frac{1}{4}</math>, <math>\frac{2}{4}</math> and <math>\frac{3}{4}</math> of a quantity.</li> </ul>	
<p><b>Progression of skills</b></p>	<p><b>Key representations</b></p>	
<p><b>Divide by 2</b> Encourage children to compare the grouping and sharing structures of division and to make links with times-table facts and halving.</p>	<p>There are ... equal groups of 2 ... <math>\div 2 = \dots</math></p>  <p><math>4 \times 2 = 8</math> <math>8 \div 2 = 4</math></p>	<p>... shared equally between 2 is ... Half of ... is ... ... <math>\div 2 = \dots</math></p>  <p><math>4 \times 2 = 8</math> <math>8 \div 2 = 4</math></p>
<p><b>Divide by 10</b> Encourage children to compare the grouping and sharing structures of division and to make links with times-table facts.</p>	<p>There are ... equal groups of 10 ... <math>\div 10 = \dots</math></p>  <p><math>6 \times 10 = 60</math> <math>60 \div 10 = 6</math></p>	<p>... shared equally between 10 is ... ... <math>\div 10 = \dots</math></p>  <p><math>6 \times 10 = 60</math> <math>60 \div 10 = 6</math></p>
<p><b>Divide by 5</b> Encourage children to compare the grouping and sharing structures of division and to make links with times-table facts.</p>	<p>There are ... equal groups of 5 ... <math>\div 5 = \dots</math></p>  <p><math>6 \times 5 = 30</math> <math>30 \div 5 = 6</math></p>	<p>... shared equally between 5 is ... ... <math>\div 5 = \dots</math></p>  <p><math>6 \times 5 = 30</math> <math>30 \div 5 = 6</math></p>

**Missing numbers**

Bar models are useful to show the link between multiplication and division.

... divided by 2/5/10 is equal to ...

**Unit fractions**

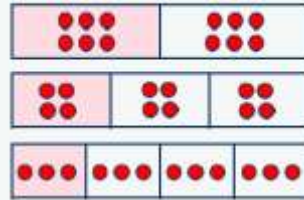
In Y2 the focus is on finding  $1/2$ ,  $1/4$  and  $1/3$

Bar models are useful to show the link between division and finding a fraction.

The objects have been shared fairly into ...

groups.

$\frac{1}{\square}$  of ... is ...



There are ... equal parts.

There is ... part circled.

$\frac{1}{\square}$  is circled.

**Non-unit fractions**

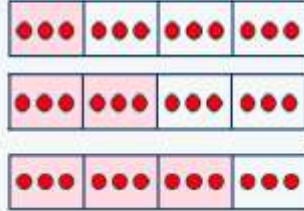
In Y2, the focus is on finding  $2/4$  and  $3/4$

Prompt children to notice that  $2/4$  is equivalent to  $1/2$

The objects have been shared fairly into ...

groups.

$\frac{\square}{\square}$  of ... is ...



There are ... equal parts.

There are ... parts circled.

$\frac{\square}{\square}$  is circled.




<b>Year 3</b>	<ul style="list-style-type: none"> <li>Recall and use division facts for the 3, 4 and 8 multiplication tables.</li> <li>Write and calculate mathematical statements for division using the multiplication tables that they know, including for two-digit numbers times one-digit numbers, using mental and progressing to formal written methods.</li> <li>Recognise, find and write fractions of a discrete set of objects: unit fractions and non-unit fractions with small denominators.</li> </ul>
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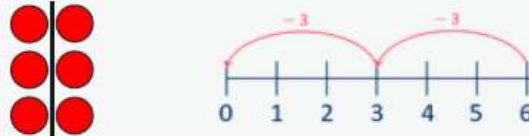
<b>Progression of skills</b>	<b>Key representations</b>
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**Divide by 3**  
Encourage children to compare the grouping and sharing structures of division and to make links with times-table facts.

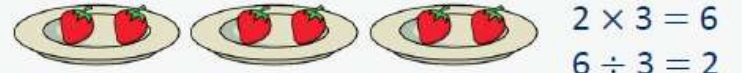
There are ... groups of 3 in ...  
...  $\div 3 =$



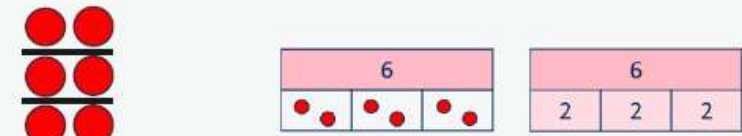
$2 \times 3 = 6$   
 $6 \div 3 = 2$



... has been shared equally into 3 equal groups.  
...  $\div 3 =$




$2 \times 3 = 6$   
 $6 \div 3 = 2$

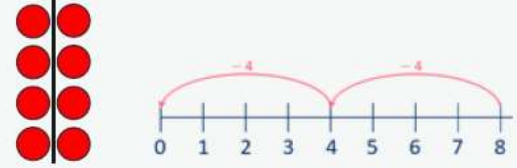


**Divide by 4**  
Encourage children to compare the grouping and sharing structures of division and to make links with times-table facts.

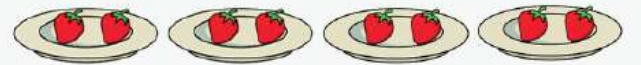
There are ... groups of 4 in ...  
...  $\div 4 =$



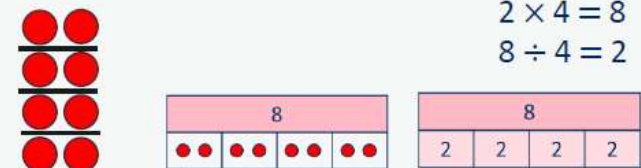
$2 \times 4 = 8$   
 $8 \div 4 = 2$



... has been shared equally into 4 equal groups.  
...  $\div 4 =$



$2 \times 4 = 8$   
 $8 \div 4 = 2$



**Divide by 8**  
Encourage children to compare the grouping and sharing structures of division and to make links with times-table facts.

There are ... groups of 8 in ...  
...  $\div$  8 =

$2 \times 8 = 16$   
 $16 \div 8 = 2$

... has been shared equally into 8 equal groups.  
...  $\div$  8 =

$2 \times 8 = 16$   
 $16 \div 8 = 2$

**Related facts**  
Link to known times-table facts.

...  $\div$  ... is equal to ...,  
so ... tens  $\div$  ... is equal to ... tens.

$12 \div 3 = 4$   
 $120 \div 3 = 40$

**Divide a 2-digit number by a 1-digit number - no exchange**  
Partition into tens and ones to divide and then recombine.

... tens divided by ... is equal to ... tens.  
... ones divided by ... is equal to ... ones.

Tens	Ones

$60 \div 2 = 30$   
 $4 \div 2 = 2$

$64 \div 2 = 32$

Tens	Ones

**Divide a 2-digit number by a 1-digit number - with remainders**  
Encourage children to partition numbers flexibly to help them to divide more efficiently.

... tens divided by ... is equal to ... tens.  
... ones divided by ... is equal to ... ones.

Tens	Ones

$96 \div 4$

$80 \div 4 = 20$   
 $16 \div 4 = 4$   
 $96 \div 4 = 24$

There are ... groups of ...  
There are ... remaining.

$31 \div 4 = 7 \text{ r}3$

$94 \div 4 = 23 \text{ r}2$

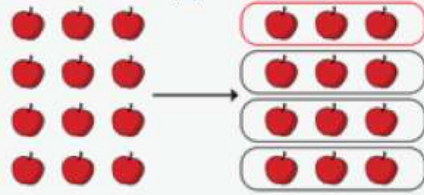
Tens	Ones

**Unit fractions of a set of objects**

Bar models are useful to show the link between division and fractions, for example, dividing by 3 and finding a third.

The whole is divided into ... equal parts.

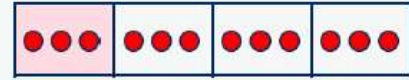
Each part is  $\frac{1}{\square}$  of the whole.



$\frac{1}{4}$  of 12 apples is 3 apples.

One ... of ... is ...

$\frac{1}{4}$  of 12 is 3



$\frac{1}{3}$  of 36 is 12

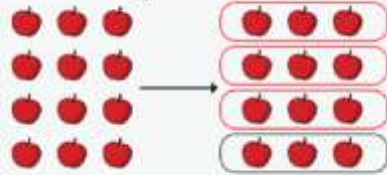


**Non-unit fractions of a set of objects**

Bar models are a useful representation and show the links with division and multiplication.

The whole is divided into ... equal parts.

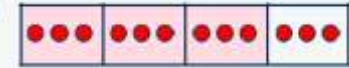
Each part is  $\frac{1}{\square}$  of the whole.



$\frac{3}{4}$  of 12 apples is 9 apples.

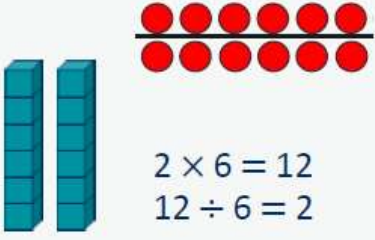
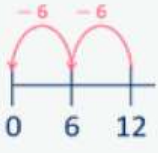
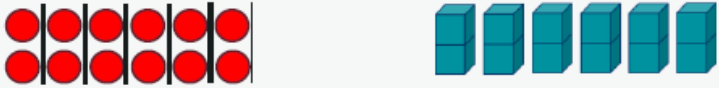





$\frac{1}{\square}$  of ... is ..., so  $\frac{\square}{\square}$  of ... is ...

$\frac{3}{4}$  of 12 is 9



$\frac{2}{3}$  of 36 is 24

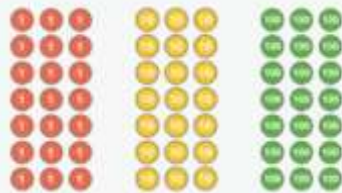


<b>Year 4</b>	<ul style="list-style-type: none"> <li>Recall division facts for multiplication tables up to <math>12 \times 12</math></li> <li>Use place value, known and derived facts to divide mentally, including: dividing by 1</li> <li>Find the effect of dividing a one- or two-digit number by 10 and 100, identifying the value of the digits in the answer as ones, tenths and hundredths.</li> </ul>	
<b>Progression of skills</b>	<b>Key representations</b>	
<b>Division facts to <math>12 \times 12</math></b> Encourage children to compare the grouping and sharing structures of division and to make links with times-table facts.	<p>There are ... groups of ... in ...  <math>\dots \div \dots =</math></p>  <p><math>2 \times 6 = 12</math>  <math>12 \div 6 = 2</math></p> 	<p>... has been shared equally into ... equal groups.  <math>\dots \div \dots =</math></p>   <p><math>2 \times 6 = 12</math>  <math>12 \div 6 = 2</math></p>
<b>Divide a number by 1 and itself</b> Children may try to divide a number by zero and it should be highlighted that this is not possible.	<p>When I divide a number by 1, the number remains the same.</p> <p>5 shared between 1 is 5 </p> <p>There are 5 groups of 1 in 5</p> 	<p>When I divide a number by itself, the answer is 1</p> <p>5 shared between 5 is 1</p>  <p>There is 1 group of 5 in 5</p> 

**Related facts**

Link to known times-table facts.

... ÷ ... is equal to ...  
 so ... tens ÷ ... is equal to ... tens  
 and ... hundreds ÷ ... is equal to ... hundreds.

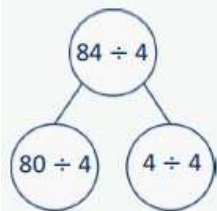


$$\begin{array}{l} 21 \div 7 = 3 \\ 210 \div 7 = 30 \\ 2,100 \div 7 = 300 \end{array} \qquad \begin{array}{l} 21 \div 3 = 7 \\ 210 \div 3 = 70 \\ 2,100 \div 3 = 700 \end{array}$$

**Divide a 2 or 3-digit number by a 1-digit number**

Progress from divisions with no exchange, to divisions with exchange and then divisions with remainders.

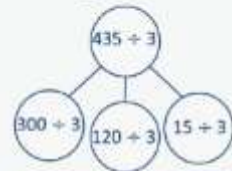
I can partition ... into ... tens and ... ones.



$$\begin{array}{l} 80 \div 4 = 20 \\ 4 \div 4 = 1 \\ 84 \div 4 = 21 \end{array}$$

Tens	Ones
●●	●
●●	●
●●	●
●●	●

I cannot share the hundreds/tens equally, so I need to exchange 1 ... for 10 ...



$$\begin{array}{l} 300 \div 3 = 100 \\ 120 \div 3 = 40 \\ 15 \div 3 = 5 \\ 435 \div 3 = 145 \end{array}$$

Hundreds	Tens	Ones
●	●●●	●●●●●
●	●●●	●●●●●
●	●●●	●●●●●
●	●●●	●●●●●

	1	3
3	3	9

T	O
●●●	●●●●●
	●●●●●
	●●●●●

**Divide by 10 and 100**

Encourage children to notice that dividing by 100 is the same as dividing by 10 twice.

When I divide by 10, the digits move 1 place value column to the right.  
 ... is one-tenth the size of ...

O	Tth	Hth
●●		
O	Tth	Hth
	●●	
2 ÷ 10 = 0.2		

T	O	Tth	Hth
●	●●●		
T	O	Tth	Hth
	●	●●●	
12 ÷ 10 = 1.2			

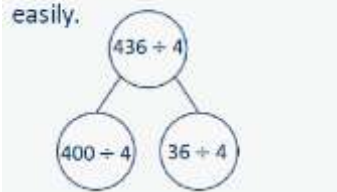
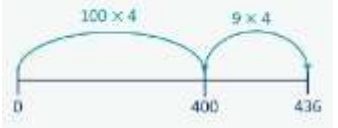
When I divide by 100, the digits move 2 place value columns to the right.  
 ... is one-hundredth the size of ...

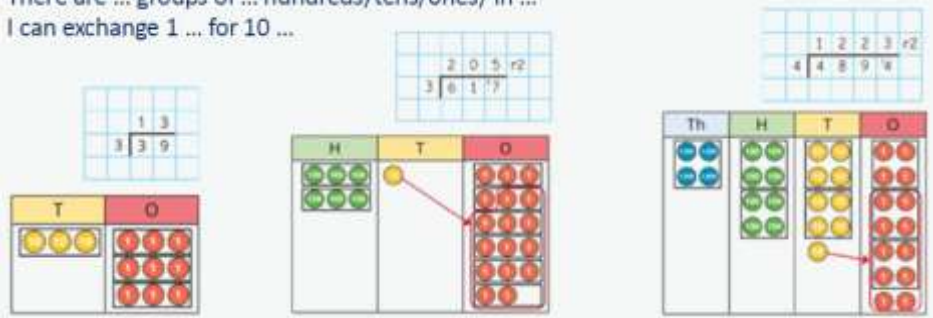
O	Tth	Hth
●●		
O	Tth	Hth
		●●
2 ÷ 100 = 0.02		

T	O	Tth	Hth
●	●●●		
T	O	Tth	Hth
		●	●●●
12 ÷ 100 = 0.12			

Year 5	<ul style="list-style-type: none"> <li>• Divide numbers mentally drawing upon known facts.</li> <li>• Divide numbers up to 4 digits by a one-digit number using the formal written method of short division and interpret remainders appropriately for the context.</li> <li>• Divide whole numbers and those involving decimals by 10, 100 and 1,000</li> </ul>
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Progression of skills	Key representations		
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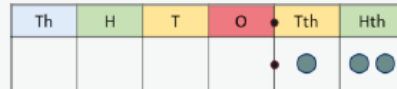
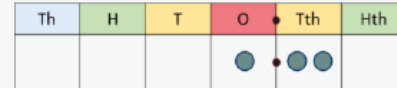
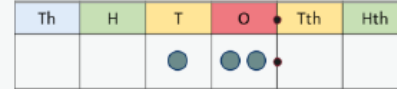
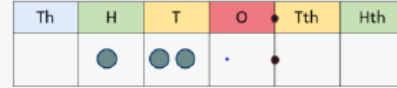
Mental strategies	<p>I can partition ... into ... and ... to help me to divide more easily.</p> 	<p>I can show groups of ... on a number line.</p> 	<p>To divide by ..., I can divide by ... and then divide the result by ...</p> $436 \div 4 = 436 \div 2 \div 2$ $436 \div 2 = 218$ $218 \div 2 = 109$
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<p><b>Divide numbers up to 4 digits by a 1-digit number</b></p> <p>The short division method is introduced for the first time.</p>	<p>There are ... groups of ... hundreds/tens/ones/ in ... I can exchange 1 ... for 10 ...</p> 		
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**Divide by 10, 100 and 1,000**

Encourage children to notice that dividing by 100 is the same as dividing by 10 twice, and that dividing by 1,000 is the same as dividing by 10 three times.

To divide by 10/100/1,000, I move all the digits ... places to the right.  
... is one-tenth/one-hundredth/one-thousandth the size of ...



$$120 \div 10 = 12$$

$$120 \div 100 = 1.2$$

$$120 \div 1,000 = 0.12$$

**Fraction of an amount**

Bar models support children to understand that to find a fraction of an amount, we divide by the denominator and multiply by the numerator.

To find  $\frac{\square}{\square}$  of ... , I need to divide by ... and multiply by ...



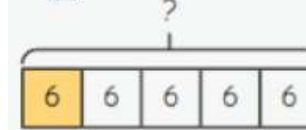
$$\frac{1}{5} \text{ of } 20 =$$

$$\frac{1}{4} \text{ of } 84 =$$

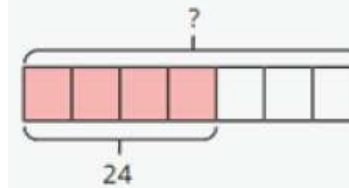
$$\frac{3}{5} \text{ of } 20 =$$

$$\frac{3}{4} \text{ of } 84 =$$

If  $\frac{1}{\square}$  is ... , then the whole is ...  $\times$  ...



$$\frac{1}{5} \text{ of } \underline{\quad} = 6$$



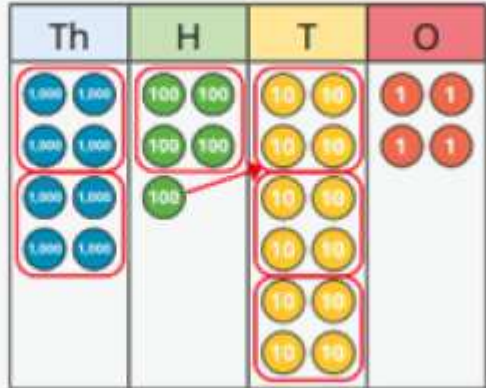
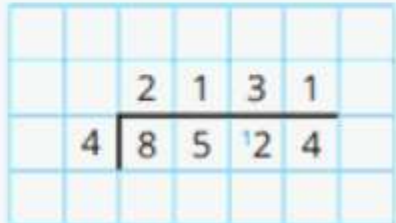
$$\frac{4}{7} \text{ of } \underline{\quad} = 24$$

<b>Year 6</b>	<ul style="list-style-type: none"> <li>• Perform mental calculations, including with mixed operations and large numbers.</li> <li>• Divide numbers up to 4 digits by a two-digit whole number using the formal written method of long division, and interpret remainders as whole number remainders, fractions, or by rounding, as appropriate for the context.</li> <li>• Divide numbers up to 4 digits by a two-digit number using the formal written method of short division where appropriate, interpreting remainders according to the context.</li> <li>• Divide numbers by 10, 100 and 1,000 giving answers up to three decimal places.</li> <li>• Use written division methods in cases where the answer has up to two decimal places.</li> <li>• Associate a fraction with division and calculate decimal fraction equivalents.</li> <li>• Divide proper fractions by whole numbers [for example, <math>1 \div 2 = \frac{1}{2}</math>]</li> <li>• Solve problems involving the calculation of percentages.</li> </ul>
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<b>Progression of skills</b>	<b>Key representations</b>
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**Short division**  
Encourage children to interpret remainders in context, for example knowing that "remainder 1" could mean complete boxes with 1 left over so 5 boxes will be needed.

There are ... groups of ... hundreds/tens/ones/ in ...  
I can exchange 1 ... for 10 ...

**Mental strategies**  
Include partitioning and number line strategies outlined in Y5 as well as division using factors.

To divide by ... , I can first divide by ... and then divide the answer by ...

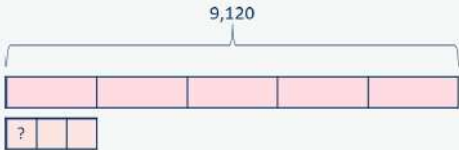
$240 \div 60 = 240 \div 10 \div 6$

240 →  $\div 10$  →  →  $\div 6$  →

$480 \div 24 = 480 \div 4 \div 6$

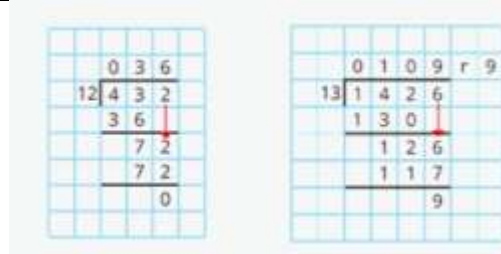
480 →  $\div 4$  →  →  $\div 6$  →

$9,120 \div 15 = 9,120 \div 5 \div 3$



**Long division**

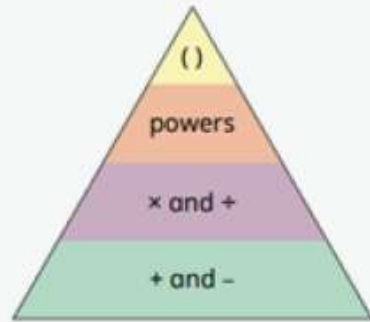
The long division method is introduced for the first time. Two alternative methods are shown.



**Order of operations**

Calculations in brackets should be done first, then powers. Multiplication and division should be performed before addition and subtraction.

... has greater priority than ..., so the first part of the calculation I need to do is ...



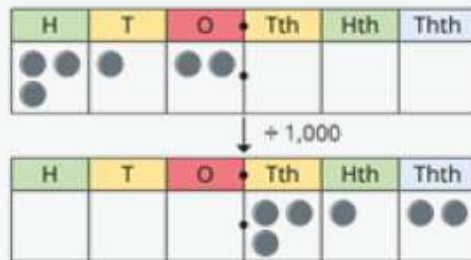
$(6 + 4) \div 2 = 5$

$6 + 4 \div 2 = 8$

**Divide by 10, 100 and 1,000**

Encourage children to notice that dividing by 100 is the same as dividing by 10 twice, and that dividing by 1,000 is the same as dividing by 10 three times.

To divide by ... , I move the digits ... places to the right.



$312 \div 10 = 31.2$   
 $312 \div 100 = 3.12$   
 $312 \div 1,000 = 0.312$

$906 \div 10 = 90.6$   
 $906 \div 100 = 9.06$   
 $906 \div 1,000 = 0.906$

**Divide decimals by integers**  
 This is the first time children divide decimals by numbers other than 10, 100 or 1,000

I know that ... ÷ ... = ...,  
 so I also know that ... ÷ ... = ...



$$39 \div 3 = 13$$

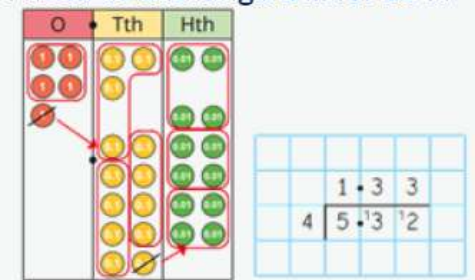


$$3.9 \div 3 = 1.3$$

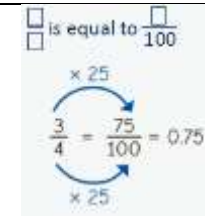
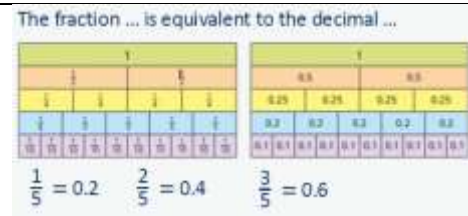


$$0.39 \div 3 = 0.13$$

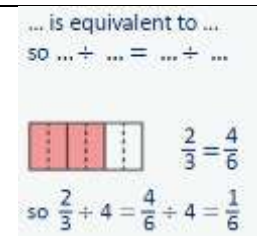
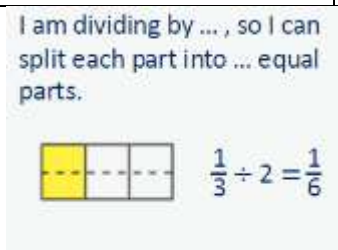
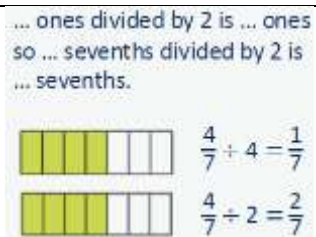
I need to exchange 1 ... for 10 ...



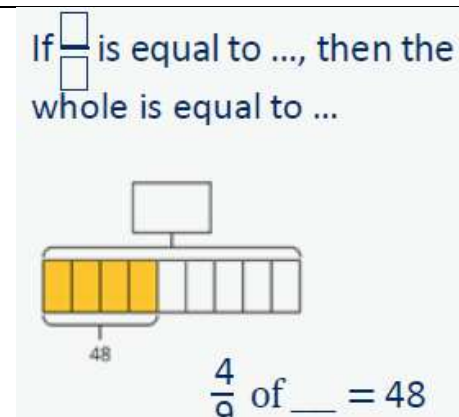
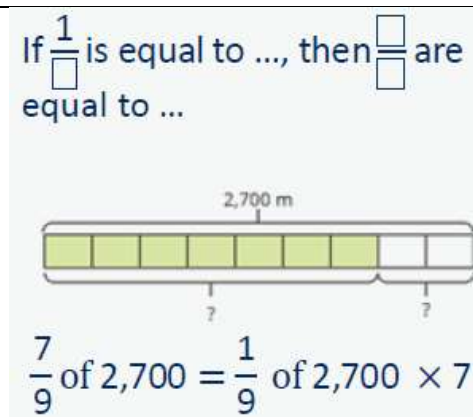
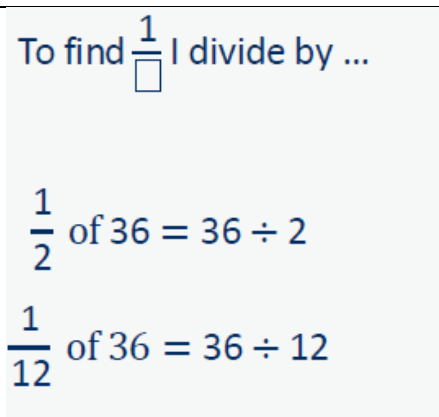
**Decimal and fraction equivalents**



**Divide a fraction by an integer**  
 This is the first time children divide fractions by an integer.



**Fraction of an amount**  
 Children divide and multiply to find fractions of an amount. Bar models can still be used to support understanding where needed.



**Calculate percentages**

Children first learn how to find 1%, 10%, 20%, 25% and 50% before using multiples of these amounts to find any percentage.

There are ... lots of ... % in 100%  
To find ... %, I need to divide by ...

100%			
50%		50%	
25%	25%	25%	25%

50% of ... = ... ÷ 2  
25% of ... = ... ÷ 4

... % is made up of ... %, and ... %

100%									
10%	10%	10%	10%	10%	10%	10%	10%	10%	10%

To find 30%, I can find 10% and then multiply it by 3  
To find 23%, I can use 10% × 2 and 1% × 3  
To find 99%, I can find 1%, then subtract from 100%

**Calculations involving ratio**

Encourage children to see the multiplicative relationship between ratios. They will need to multiply or divide each value by the same number to keep the ratio equivalent. Double number lines and ratio tables help children to see both horizontal and vertical multiplicative relationships.

For every ... , there are ...

For every 6 children on a school trip, there is 1 adult.



The ratio of children to adults is 6 : 1

Adults	Children
1	6
2	12
3	18

Annotations: A green arrow from 1 to 3 in the Adults column is labeled ÷ 6. A red arrow from 6 to 18 in the Children column is labeled ÷ 3. A green arrow from 6 to 18 in the Children column is labeled ÷ 6.

